Modeling Persistent and Periodic Weekly Rainfall in an Environment of an Emerging Sri Lankan Economy

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Abstract. The quantity of rainfall and its related events have become more and more uncertain due to climatic variability. The complexity of the rainfall pattern increases due to the changes of the atmospheric behavior from time to time. Relatively, few measures have been taken to perform the modeling of rainfall in the context of long memory. This paper provides an assessment of such a phenomenon by fitting an appropriate time series model. A long range dependency model is proposed to fit weekly rainfall data to explore characteristics of persistence through an unbounded spectral density. Careful examination of the data exhibits periodic fluctuations as an additional feature. Since, the rainfall series exhibits periodic variations and persistence, a seasonal autoregressive fractionally integrated moving average (SARFIMA) model is fitted. Parameters of it are estimated using maximum likelihood estimation (MLE) method. A Monte Carlo simulation was carried out with different seasonal and non seasonal fractionally differing parameters to measure the suitability of the method for parameter estimation. Best fitted model is chosen based on the minimum of the mean absolute error and the forecasting performance are compared with the result of Seasonal autoregressive integrated moving average (SARIMA) using an independent sample as a creative contribution.

Keywords: Seasonality · Rainfall · Fractional differencing
Long-memory · Maximum likelihood estimators · Forecasting

1 Introduction

Sri Lanka is highly vulnerable to the impacts of climate change which includes severe droughts, heavy flash floods and landslides over the past years. These whether related disasters affects many sectors in the economy and threaten to sustainable development of the country. Temporal variability of rainfall in the

V. Kreinovich and S. Sriboonchitta (Eds.): TES 2019, SCI 808, pp. 314–328, 2019.
https://doi.org/10.1007/978-3-030-04263-9_24

country is changing frequently which affect on staple food production such as rice, coconut and tea (Baba [1], Fernando et al. [17], Wejerathne et al. [41]). Sri Lanka is on a path of rapid urbanization. Thus, many damages can occur due to changes in rainfall behavior in urban areas with a high population density and modern infrastructure. Lo and Koralegedera [25] claimed that the more cities including Colombo in Sri Lanka are at a risk of water related issues due to changes in rainfall patterns. Also, each year the government of Sri Lanka spends huge amount of money to reconstruct and renovate the infrastructures which damage caused by floods in the wet zone (Sri Lanka Rapid Post Disaster Needs Assessment [37]). Accurate information on rainfall predictions enhances the ability to utilize the water resource in a productive manner. Those details of temporal variability of rainfall is not only important for agricultural activities, but also for important subject domains in urban areas such as construction, industrial planning, urban traffic and sewer systems, health, tourism, rainwater harvesting and climate monitoring. Furthermore, the importance of analysis of weekly rainfall pattern is highlighted by Silva and Peiris [34] and they analyzed weekly rainfall in Sri Lanka using percentiles bootstrap approach. Moreover, the same authors (Silva and Peiris [35]) carried out another study to explain the behavior of the south west monsoon rainfall by utilizing the weekly rainfall percentiles along with the 95% confidence interval bands using best fitted distribution for weekly rainfall in Colombo city. However, they emphasized precise rainfall prediction is very difficult in the tropical country like Sri Lanka with the low technology. According to Luk [26] also, quantitative forecasting of rainfall is extremely difficult. Silva and Peiris [36] discussed the problems faced when analyzing rainfall amount which has heavy skewed distribution using Weibull confidence interval for rainfall percentiles. The main goal of this study is to suggest a best fit long-memory model to capture weekly rainfall behavior. SARFIMA model is utilized in such a context. The outline of this paper is as follows. Section 2 describes past works related to long memory models. The functional form of the SARFIMA model with the maximum likelihood estimation procedure is described by Sect. 3. It is followed by Sect. 4 that will present the Monte Carlo simulation results to assess the accuracy and reliability of the estimation procedure. Section 5 will provide results of weekly rainfall modeling. Finally, Sect. 6 provides some concluding remarks.

2 Literature Related to Long Memory Models

Time series models have been developed with an increasing degree of accuracy over the last few decades. The short memory autoregressive moving average (ARMA) model introduced by Box and Jenkins [5] has been extensively used for a variety of applications. Recently, time series models with long memory features became very popular among researchers in many fields such as statistics and econometrics. Features of a fractionally integrated autoregressive moving average (ARFIMA) long memory model was initially introduced by Granger and Joyeux [20] and Hosking [24]. It was an extension of the traditional ARMA process with a fractional differencing parameter. The hyperbolic decay of the autocorrelation function and an unbounded spectral density are two key features of the ARFIMA process. A number of estimation methods of the fractional differencing parameter were proposed by Porter-Hudak and Geweke [19], Fox and Taqqu [18], Dahlhaus [10], Sowell [38], Chen et al. [7] and Robinson [33]. Comparison study assessments were done by Cheung and Diebold [8] on maximum likelihood estimators for fractionally differenced parameters using two types of maximum likelihood (ML) estimators in the form of frequency-domain ML and exact domain ML of time series processes with an unknown mean. Small sample properties of four ML estimators of the ARFIMA model was investigated through a Monte Carlo simulation by Hauser [23]. A study done by Wang et al. [40] evaluated the ability of detecting existence of long-memory in time series using four methods: Lo's modified rescale adjusted range test, Geweke and Porter Hudak test and two other approximate maximum likelihood estimation methods. Some of the research done by Chan and Palma [6], Palma [28] and Beran et al. [3] carried out an assessment of ARFIMA model parameters and their properties. Dissanayake [11] introduced a rapid lag order detection mechanism of the standard long memory ARFIMA process. Due to the practical success of the ARFIMA model, a more generalized fractionally differenced long memory time series model called the Gegenbauer ARMA (GARMA) was probed in detail by Gray et al. [21]. Chung [9] extended the work in introducing a grid based parameter estimation procedure of an elementary GARMA process. Fresh interest in the econometric community infused into the process the introduction of a new class of models with heteroskedasticity in Dissanayake and Peiris [12]. It was followed by the casting of the process driven by Gaussian white noise in state space by Dissanayake et al. [13] to establish a parameter estimation based optimal lag order validated by predictive accuracy. A similar experiment in which the process was driven by Generalized Autoregressive Conditionally heteroskedastic (GARCH) errors (instead of Gaussian white noise) was presented in Dissanayake et al. [14] with the validation of parameter estimation based optimal lag order done through log likelihood measures. A concise summary of fractionally differenced Gegenbauer processes with long memory was provided in Dissanayake [15]. An extensive review of fractionally differenced Gegenbauer processes with long memory is found in Dissanayake et al. [16]. It refers to certain conceptual paradigms presented in the survey on long memory by Guegan [22] in which an extended k-factor Gegenbauer process becomes a highlight of rigour. Though the ARFIMA model was able to capture the long range dependency, it does not take into account the seasonal variation patterns present in some real data set. The seasonal autoregressive fractionally integrated moving average (SARFIMA) of Porter-Hudak [30] is a natural extension of the ARFIMA process with an additional seasonal filter. The model consists of long memory dependency features with periodic behavior in terms of the data. SARFIMA model was utilized for forecasting of the monthly IBM product revenue in Ray [32]. Peiris and Singh [29] suggested a convenient method to calculate predictors for seasonal and non seasonal fractional parameters of long memory models under certain conditions. The work done by Bisognin and Lopes [4] described number of properties of seasonally fractional ARMA process in detail. SARFIMA model was applied to forecast Iraqi oil production and model parameters were estimated using conditional sum of squares by Mostafaei and Sakhabakhsh [27]. Additionally, Reisen et al. [31] proposed a semi parametric approach to estimate two seasonal fractional parameters in a SARFIMA model and the performance was evaluated through a Monte Carlo experiment. Very few attempts have been made to study the rainfall behavior in context of long memory. A study done by Yaya and Fashae [42] made an attempt to fit SARFIMA models for rainfall data in six rainfall zones of Nigeria. However, they could not find significant SARFIMA models which can capture the seasonal behavior with the long range dependency of the real data. Utilizing a SARFIMA model to assess seasonal and persistent properties of weekly rainfall in an emerging Asian economy such as Sri Lanka is missing in the current literature. Theoretical concepts linked with the long memory SARFIMA model are provided in the next section.

3 SARFIMA Long Memory Model

Long range dependency features can be identified by two different approaches but equivalent forms given below defined in two distinct domains called time and frequency (Bary [2]). In time domain, the auto correlation function $\rho_X(.)$ of the time series decays hyperbolically to zero. The correlation function, $\rho_X(k) \approx k^{2d-1}$ when $k \to \infty$ and $0.0 < d < 0.5$. The frequency domain, spectral density function $f_X(.)$ is unbounded when the frequency is near zero, that is, $f_X(\omega) \approx \omega^{-2d}$ when $\omega \to 0$. A process $\{Y_t\}_{t \in Z}$ is a stationary stochastic process given by the formula,

$$\phi(B)\psi(B^S)\nabla^d\nabla_S^D(Y_t - \mu) = \theta(B)\Theta(B^S)\epsilon_t \tag{1}$$

where μ is the mean of the process, $\{\epsilon_t\}_{t \in Z}$ is a white noise process with zero mean and variance σ_ϵ^2. B is the backward shift operator such that $y_{t-n} = B^n y_t$ and s is the seasonal length. $\phi(B)$ and $\psi(B)$ are the non seasonal and seasonal autoregressive polynomials of order p and P respectively such that

$$\phi(B) = \sum_{i=1}^{p} \phi_i B^i \quad 1 \le i \le p \quad \psi(B) = \sum_{k=1}^{P} \psi_k B^k \quad 1 \le k \le P \tag{2}$$

$\theta(B)$ and $\Theta(B)$ are the non-seasonal and seasonal moving average polynomials of order q and Q respectively defined as

$$\theta(B) = \sum_{j=1}^{q} \theta_j B^j \quad 1 \le j \le q \quad \Theta(B) = \sum_{m=1}^{Q} \Theta_m B^m \quad 1 \le m \le Q \tag{3}$$

The differencing operator ∇^d can be expressed as,

$$\nabla^d = (1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k \tag{4}$$

where $\binom{d}{k} = \frac{\Gamma(1+d)}{\Gamma(1+k)\Gamma(1+d-k)}$.

The seasonal operator ∇_S^D can be expressed as,

$$\nabla_S^D = (1 - B^S)^D = \sum_{k=0}^{\infty} \binom{D}{k} (-B^S)^k \tag{5}$$

where $\binom{D}{k} = \frac{\Gamma(1+D)}{\Gamma(1+k)\Gamma(1+D-k)}$.

The model (1) is specified by $SARFIMA(p,d,q)x(P,D,Q)_S$. When $d = 0$ and $D = 0$, the model is reduced to a classical seasonal SARFIMA model. If conditions 0 < d < 0.5 and 0 < D < 0.5 are satisfied the process becomes stationary. The spectral density function of the SARFIMA model can be written as follows:

$$f_S(\lambda) = \frac{\sigma_\epsilon^2 |\theta_q e^{(-i\lambda)}|^2 |\Theta_Q e^{(-i\lambda)S}|^2}{2\pi |\phi_p e^{(-i\lambda)}|^2 |\psi_P e^{(-i\lambda)S}|^2} |1 - e^{-i\lambda}|^{-2d} |1 - e^{-i\lambda S}|^{-2D} \tag{6}$$

An unbounded spectral density around the origin assessed through the utilization of (6) coupled with the hyperbolic decay of autocorrelation and partial autocorrelation blended with seasonality features (as shown in Figs. 2, 3 and 4) prompted towards parameter assessment being done using the exact maximum likelihood estimation method. It was done by maximizing the log likelihood function numerically. The package arfima in R was used to calculate the maximum likelihood estimators. Durbin-Levinson and Trench algorithms were utilized to maximize the likelihood and obtain optimal simulation and forecasting results (Veenstra and McLeod [39]).

4 Results of Monte Carlo Simulation

In order to evaluate the performance of the maximum likelihood method in estimating the parameters of the model, a number of Monte Carlo experiments were carried out. The simulation results provided non-seasonally and seasonally differenced parameter estimations and the corresponding standard and mean square errors (MSE) of the parameters. It was carried out based on 1000 replications with different sizes of samples ($n = 100$, $n = 200$, $n = 500$ and $n = 1000$). Seasonal length was considered as 52 corresponding to weekly rainfall. Monte Carlo experiment was conducted on a simulated $SARFIMA(0,d,0)x(0,D,0)_{52}$ series with following parameter combinations.

$d = 0.1$ and $D = 0.45$, $d = 0.15$ and $D = 0.45$, $d = 0.3$ and $D = 0.3$, $d = 0.45$ and $D = 0.10$.

The simulation was carried out using the R programming Language (Version 3.4.2) utilizing a HP11 (8 GB, 64 bit) computer. The standard errors of the estimates $SD(\hat{d})$, $SD(\hat{D})$ and mean square error of the estimates $MSE(\hat{d})$, $MSE(\hat{D})$ respectively such that:

$$SD(\hat{d}) = \sqrt{\sum_{r=1}^{R} (\hat{d}_r - \hat{d})/R}, \qquad SD(\hat{D}) = \sqrt{\sum_{r=1}^{R} (\hat{D}_r - \hat{D})/R},$$

$$MSE(\hat{d}) = \sum_{r=1}^{R} (\hat{d}_r - \hat{d})^2/R, \qquad MSE(\hat{D}) = \sum_{r=1}^{R} (\hat{D}_r - \hat{D})^2/R$$

Where $\hat{d}_r$ and $\hat{D}_r$ are the MLE of d and D for the rth replication. The value R denotes the number of replications ($R = 1000$ for the simulation).

Tables 1, 2 3 and 4 present the average of the estimated d, corresponding standard error and MSE of the estimator.

Table 1. MLE of d and D of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)_{52}$ with $d = 0.1$ and $D = 0.45$. The results are based on 1000 Monte Carlo replications

n	$\hat{d}$	$SD(\hat{d})$	$MSE(\hat{d})$	$\hat{D}$	$SD(\hat{D})$	$MSE(\hat{D})$
100	0.0333	0.0826	0.0112	0.4423	0.0160	0.0003
200	0.0674	0.0590	0.0045	0.4462	0.0112	0.0001
500	0.0860	0.0359	0.0014	0.4475	0.0091	0.00008
1000	0.0927	0.0297	0.0009	0.4503	0.0118	0.0001

Table 2. MLE of d and D of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)_{52}$ with $d = 0.15$ and $D = 0.45$. The results are based on 1000 Monte Carlo replications

n	$\hat{d}$	$SD(\hat{d})$	$MSE(\hat{d})$	$\hat{D}$	$SD(\hat{D})$	$MSE(\hat{D})$
100	0.0671	0.0835	0.0138	0.4502	0.0136	0.0001
200	0.1033	0.0570	0.0054	0.4539	0.0087	0.00009
500	0.1358	0.0357	0.0014	0.4530	0.0076	0.00006
1000	0.1429	0.0260	0.0007	0.4516	0.0070	0.00005

It can be seen from Tables 1, 2, 3 and 4 a reasonable assessment of the maximum likelihood estimator for the seasonal as well as non-seasonal fractional differencing parameters. It is noticeable that the parameter bias has decreased as with the increase of the series length. Also, it is evident from the parameters in Tables 1, 2, 3 and 4 that they become consistent with the increase in series length. Standard deviation and the MSE of estimators decrease with the increase in series length as expected.

Table 3. MLE of d and D of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)_{52}$ with $d = 0.3$ and $D = 0.3$. The results are based on 1000 Monte Carlo replications

n	$\hat{d}$	$SD(\hat{d})$	$MSE(\hat{d})$	$\hat{D}$	$SD(\hat{D})$	$MSE(\hat{D})$
100	0.2236	0.0831	0.0127	0.2738	0.0690	0.0127
200	0.2607	0.0586	0.0049	0.2890	0.0386	0.0016
500	0.2846	0.0360	0.0015	0.2947	0.0260	0.0007
1000	0.2907	0.0262	0.0007	0.2978	0.0190	0.0003

Table 4. MLE of d and D of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)_{52}$ with $d = 0.45$ and $D = 0.1$. The results are based on 1000 Monte Carlo replications

n	$\hat{d}$	$SD(\hat{d})$	$MSE(\hat{d})$	$\hat{D}$	$SD(\hat{D})$	$MSE(\hat{D})$
100	0.3697	0.0733	0.0118	0.0205	0.1414	0.0263
200	0.4015	0.0493	0.0047	0.0723	0.0667	0.0052
500	0.4283	0.0312	0.0014	0.0902	0.0365	0.0014
1000	0.4391	0.0244	0.0007	0.0936	0.0268	0.0007

5 Application for Real Data

5.1 Description of Dataset

Colombo city is the commercial capital of Sri Lanka, situated with latitudes 6 55 N and Longitude 79 51 E and is chosen as the study site. Daily rainfall data of Colombo were collected from 1990 to 2015 from the Department of Meteorology, Sri Lanka for this study. The daily rainfall (mm) data has been converted into weekly rainfall by dividing a year into 52 weeks such that week 1 corresponds to 1–7 January, Week 2 corresponds to 8–14 January and so on. The data during the time span from 1990 to 2014 was used as to build the models while the rest was used for the model validation.

5.2 Model Development

To examine the temporal variability of the rainfall series, time series plots was taken and it is presented in Fig. 1.

Random behavior of the rainfall pattern can be clearly observed in Fig. 1. However, it cannot be identified as a decreasing or increasing trend in weekly rainfall within the considered time span. In order to find out the seasonal behavior of the data, autocorrelation analysis was carried out and the result is presented in Figs. 2 and 3. A seasonal behavior can be clearly recognized from this plot. Since the data was captured on a weekly basis, identifying the seasonal length, ACF and PACF were done with 52 lag difference.

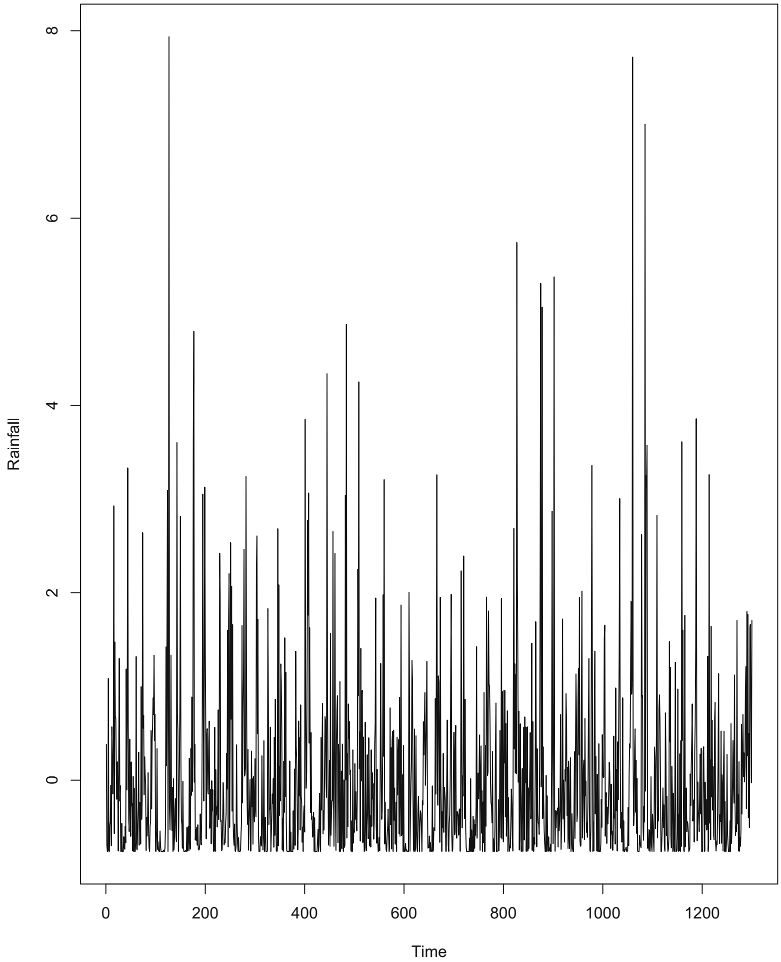

Fig. 1. Time series plot of weekly rainfall series from 1990 to 2014

Based on the ACF and PACF it can be clearly identified that the seasonal length is 52 since significant sample autocorrelation existed in the 52^{nd} Lag. Figure 5 illustrate the sample spectrum which has the peak at frequency very closer to zero. The corresponding frequency gives a maximum spectrum density of 0.0385185. This value is not far from zero. (0.0385185 * 100/0.5 = 0.0770 = 7.7%). Thus, we can conclude that the SARFIMA series is suitable for this data set. The long term serial correlation in the data are accounted for in long memory modeling. It is very imperative to consider the long memory features to capture the real dynamics of rainfall. Thus, several SARFIMA models were fitted to the data with the size of the sample being 1300. Those fitted were utilized to predict the weekly rainfall over the year 2015. The best fitted model is

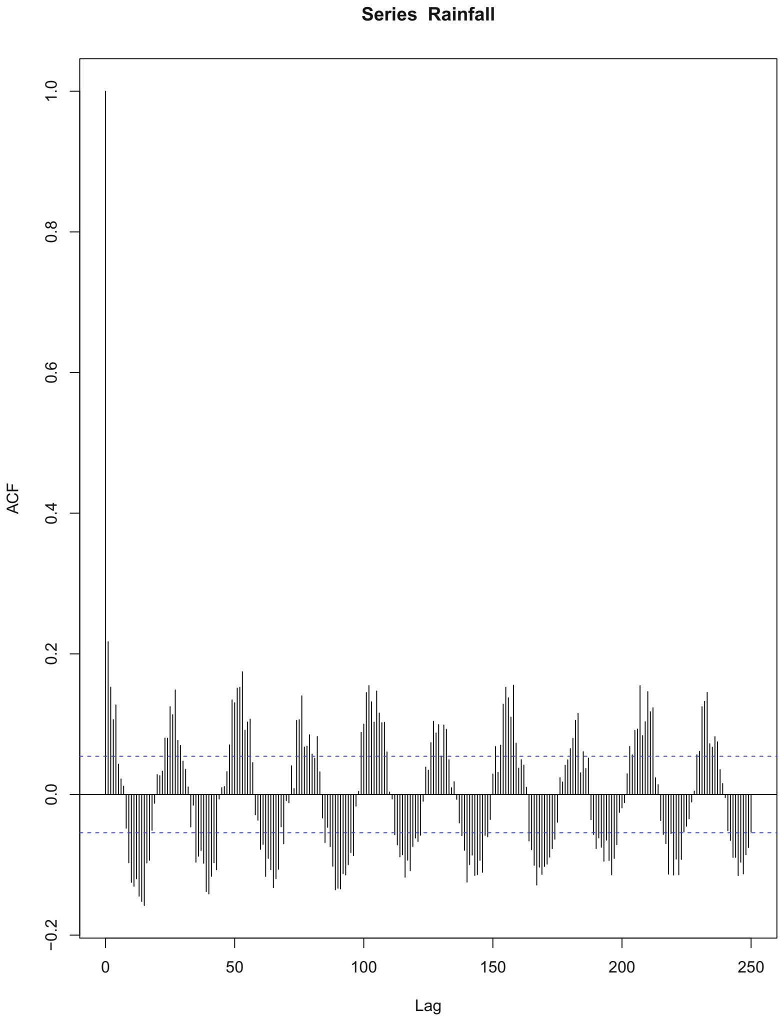

Fig. 2. Autocorrelation plot of the series from 1990 to 2014

selected with minimum mean absolute error (MAE). The MAE can be written as;

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |e_i|.$$

Where e_i is the forecasting error and n is the length of the forecasting series. The corresponding result of the fitted long memory model is as follows.
A model $SARFIMA\ (1, 0.116, 1) \times (1, 0.171, 0)_{52}$ was found to be the best fitted model for the weekly rainfall series. The corresponding parameter estimates with standard errors are presented in Table 5.

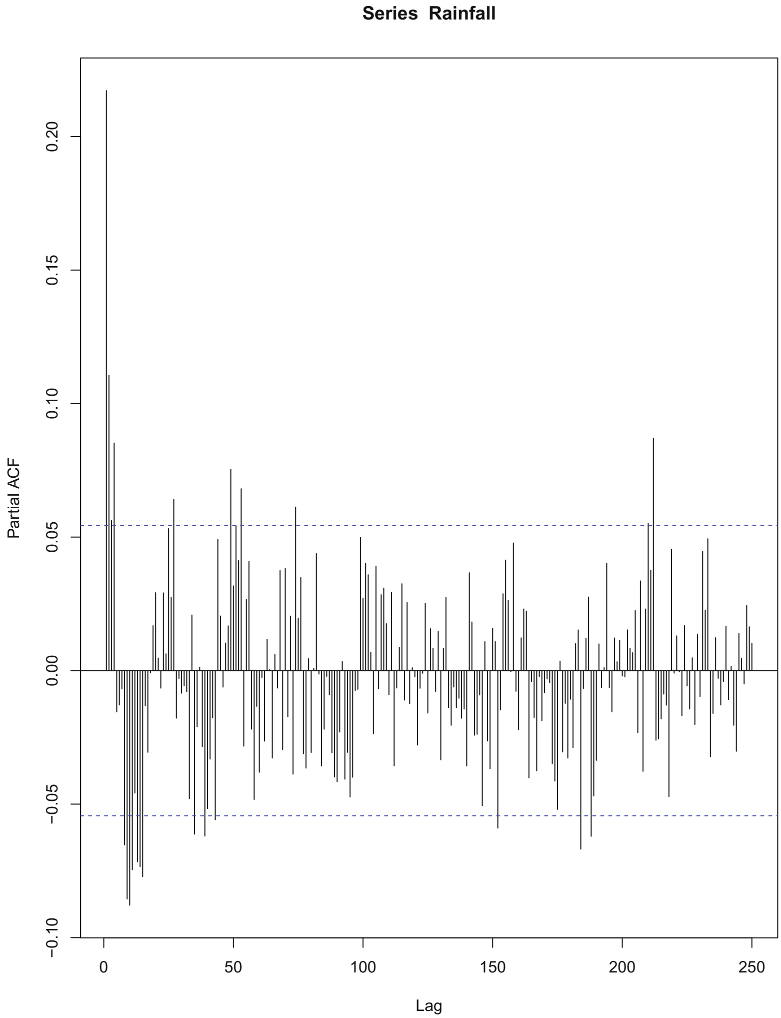

Fig. 3. Partial autocorrelation plot of the series from 1990 to 2014

Table 5. Fitted model for the weekly rainfall series $SARFIMA(p, d, q)x(P, D, Q)_S$ with $p = 1$, $q = 1$, $d = 0.1156735$, $P = 1$, $D = 0.1707546$, $Q = 0$ and $S = 52$

Coefficients	ϕ_1	θ_1	ψ_1	Constant	d	D
Estimate	−0.9113	−0.9018	−0.0860	0.0041	0.1156	0.1707
Standard Error	0.1427	0.1498	0.0394	0.1021	0.0269	0.0291
Z-value	−6.3854	−6.0192	−2.1796	0.0401	4.2900	5.8644
Pr(>|Z|)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

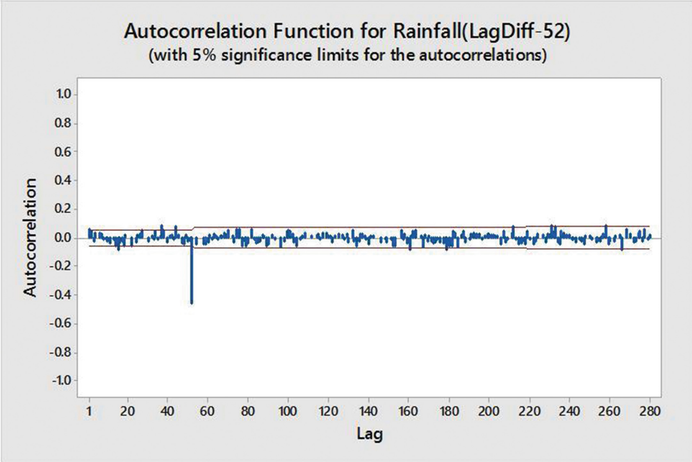

Fig. 4. ACF of the series from 1990 to 2014 with 52 Lag

All model parameters are significant at the 0.05 level of significance. The residual analysis of the fitted model was performed and found the uncorrelated at a 5% level of significance. Furthermore, the model was tested for weekly rainfall data in 2015 and the result is presented in Table 6. Various SARIMA models also fitted to the same dataset for the purpose of the comparison with the long memory model SARFIMA. A model $SARIMA(1,0,0)x(1,0,1)_{52}$ was found to be the best fitted model for the weekly rainfall series.

Table 6. Absolute Forecasting Error in mm for independent sample 2015

Absolute Forecasting Error in mm	SARIMA weekly percentage	SARFIMA weekly percentage
0–10	7(13.5)	12(23.1)
11–15	5(9.6)	4(7.7)
16–20	3(5.8)	4(7.7)
21–25	4(7.7)	5(9.6)
26–30	2(3.8)	6(11.5)
31–35	7(13.5)	4(7.7)
36–40	8(15.4)	3(5.8)
41–45	4(7.7)	1(1.9)
46–50	1(1.9)	3(5.8)
More than 50	11(21.1)	10(19.2)

Based on the above forecasted result, the model SARFIMA is outperform SARIMA. It can be seen that the more than 30% of the weeks' forecasting error

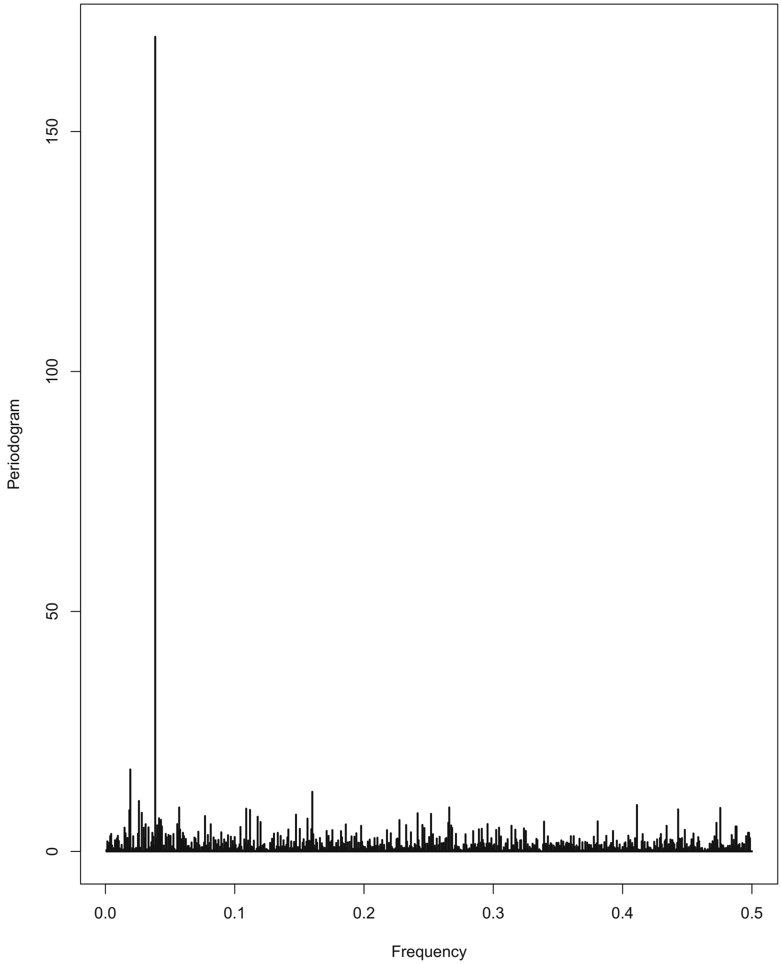

Fig. 5. The periodogram of the rainfall series from 1990 to 2014

less than to 15 mm indicated that a good agreement of the real and forecasted which derived from the SARFIMA. Then the novel model can be considered as best fitted for the weekly rainfall series. However, still there is a considerable number of weeks' forecasting error more than to 50 mm which need further improvement to this model.

6 Conclusion

It is evident from the results of this paper that long range dependency characteristics could couple with periodic variation in a weekly rainfall series. SARFIMA model can be considered to capture both long memory and seasonality. The

Monte Carlo simulation provides evidence towards optimal accuracy of parameter estimation. Furthermore, accuracy of the estimators improved with increasing of the series length. The effectiveness of the SARFIMA model was represented by using real datasets in the form of weekly rainfall from 1990 to 2014 (1300 size of the sample). $SARFIMA$ $(1, 0.116, 1) \times (1, 0.171, 0)_{52}$ model was found to be the best model to forecast weekly rainfall in Colombo city. Thereafter, model was used to make independent sample long-range seasonal predictions of the weekly rainfall for the year 2015. Those result was compared with the $SARIMA(1, 0, 1)x(1, 0, 1)_{52}$ and found that the SARFIMA is superior than the SARIMA based on the predicted performance which has done for the independent data set.

Acknowledgement. This study was partially funded by the University Research Grant, University of Sri Jayewardenepura, Sri Lanka under Grant ASP/01/RE/HSS/2016/75).

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