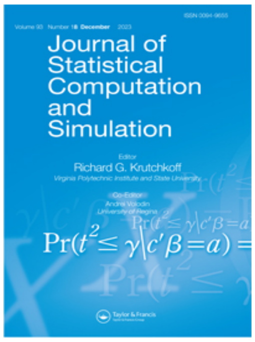

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Comparison of standard long memory time series

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ABSTRACT

Standard long memory models are in abundance in the literature today. Selecting the best such a model in terms of capturing key requisite features and trends in data becomes a challenge. This paper addresses the issue through a sequence of Monte Carlo experiments on simulated data and introduces an interval estimate on the asymptotic variance for the long-range dependence parameter of the entire family of standard long memory time series considered within the scope of the study.

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Seasonality; long memory; heteroskedasticity; filters

1. Introduction

Time series models have been developed with an increasing degree of accuracy over the last few decades. The short memory autoregressive moving average (ARMA) model introduced by Box and Jenkins [1] has been extensively used for a variety of applications. Recently, time series models with long memory features have become very popular among researchers in many fields such as statistics and econometrics. Features of a fractionally integrated autoregressive moving average (ARFIMA) long memory model were initially introduced by Granger and Joyeux [2] and Hosking [3]. It was an extension of the traditional ARMA process with a fractional differencing parameter as opposed to a autoregressive integrated moving average (ARIMA) series with a non-fractional differencing value. The hyperbolic decay of the autocorrelation function and an unbounded spectral density are two key features of the ARFIMA process. A number of estimation methods of the fractional differencing parameter were proposed by Porter-Hudak and Geweke [4], Fox and Taqqu [5], Dahlhaus [6], Sowell [7], Chen et al. [8] and Robinson [9]. Comparison study assessments were done by Cheung and Diebold [10] on maximum likelihood estimators for fractionally differenced parameters using two types of maximum likelihood (ML) estimators in the form of frequency-domain ML and exact domain ML of time series processes with an unknown mean. Small sample properties of four ML estimators of the ARFIMA model were investigated through a Monte Carlo simulation by Hauser [11]. A study done by Wang et al. [12] evaluated the ability of detecting existence of long-memory in time series using

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four methods: Lo's modified rescale adjusted range test, Geweke and Porter Hudak test and two other approximate maximum likelihood estimation methods. Some of the research done by Chan and Palma [13], Palma [14] and Beran et al. [15] carried out an assessment of ARFIMA model parameters and their properties. Dissanayake [16] introduced a rapid lag order detection mechanism of the standard long memory ARFIMA process. Due to the practical success of the ARFIMA model, a more generalized fractionally differenced long memory time series model called the Gegenbauer ARMA (GARMA) was probed in detail by Gray et al. [17]. Chung [18] extended the work in introducing a grid-based parameter estimation procedure of an elementary GARMA process. Fresh interest in the econometric community infused into the process the introduction of a new class of models with heteroskedasticity in Dissanayake and Peiris [19]. It was followed by the casting of the process driven by Gaussian white noise in state space by Dissanayake et al. [20] to establish a parameter estimation based on optimal lag order validated by predictive accuracy. A similar experiment in which the process was driven by Generalized Autoregressive Conditionally heteroskedastic (GARCH) errors (instead of Gaussian white noise) was presented in Dissanayake et al. [21] with the validation of parameter estimation based on optimal lag order done through log-likelihood measures. A concise summary of fractionally differenced Gegenbauer processes with long memory was provided in Dissanayake [22]. An extensive review of fractionally differenced Gegenbauer processes with long memory is found in Dissanayake et al. [23]. It refers to certain conceptual paradigms presented in the survey on long memory by Guegan [24] in which an extended k-factor Gegenbauer process becomes the highlight of rigour. Though the ARFIMA model was able to capture the long-range dependency, it does not take into account the seasonal variation patterns present in some of the real data sets. The seasonal autoregressive fractionally integrated moving average (SARFIMA) of Porter-Hudak [25] is a natural extension of the ARFIMA process with an additional seasonal filter. The model consists of long memory dependency features with periodic behaviour in terms of the data. Very few attempts have been made to study rainfall behaviour in the context of long memory and seasonality. A study done by Yaya and Fashae [26] made an attempt to fit SARFIMA models for rainfall data in six rainfall zones of Nigeria. However, they could not find significant SARFIMA models which can capture the seasonal behaviour with the long-range dependency of the real data. Comparative assessment of time series models that capture weekly rainfall in an emerging Asian economy such as Sri Lanka is seemingly absent in the current literature and becomes the core contribution of this article. Theoretical preliminaries linked with all such long memory models are provided in the next section. The outline of this paper is as follows. Section 2 presents theoretical notation and preliminaries related to ARFIMA and SARFIMA long memory models presented in this article. It is followed by Section 3 with a brief emphasis on methodology. Section 4 presents results of the research endeavour that are comparatively assessed in the discussion of Section 5. Finally, Section 6 provides some concluding remarks.

2. Notation and preliminaries

Long-range dependency features can be identified using two different approaches by employing equivalent forms (given below) defined in two distinct domains called time and frequency (Bary [27]). In time domain, the autocorrelation function $\rho_X(.)$ of the time series decays hyperbolically to zero. The correlation function, $\rho_X(k) \approx k^{2d-1}$ when $k \to \infty$ and

$0.0 < d < 0.5$. The frequency domain, spectral density function $f_X(.)$ is unbounded when the frequency is near zero, that is, $f_X(\omega) \approx \omega^{-2d}$ when $\omega \to 0$. Therefore $\{Y_t\}_{t\in Z}$ is a stationary stochastic process with a fractional differencing parameter d such that $0 < d < 0.5$ that defines an ARFIMA time series of the form:

$$\phi(B)\nabla^d(Y_t - \mu) = \theta(B)\epsilon_t \tag{1}$$

Adding seasonal polynomials and a seasonal filter to (1) creates the formula of a SARFIMA process as follows:

$$\phi(B)\psi(B^S)\nabla^d\nabla_S^D(Y_t - \mu) = \theta(B)\Theta(B)^S\epsilon_t \tag{2}$$

Where μ is the mean of the process, $\{\epsilon_t\}_{t\in Z}$ is a white noise process with zero mean and variance σ_ϵ^2. B is the backward shift operator such that $y_{t-n} = B^n y_t$ and s is the seasonal length. $\phi(B)$ and $\psi(B)$ are the non seasonal ad seasonal autoregressive polynomials of order p and P respectively such that

$$\phi(B) = \sum_{i=1}^{p} \phi_i B^i 1 \le i \le p \psi(B) = \sum_{k=1}^{P} \psi_k B^k 1 \le k \le P \tag{3}$$

$\theta(B)$ and $\Theta(B)$ are the non-seasonal and seasonal moving average polynomials of order q and Q respectively defined as

$$\theta(B) = \sum_{j=1}^{q} \theta_j B^j 1 \le j \le q \Theta(B) = \sum_{m=1}^{Q} \Theta_m B^m 1 \le m \le Q \tag{4}$$

The differencing operator ∇^d can be expressed as,

$$\nabla^d = (1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k \tag{5}$$

Where $\binom{d}{k} = \frac{\Gamma(1+d)}{\Gamma(1+k)\Gamma(1+d-k)}$.

The seasonal operator ∇_S^D can be expressed as,

$$\nabla_S^D = (1 - B^S)^D = \sum_{k=0}^{\infty} \binom{D}{k} (-B^S)^k \tag{6}$$

Where $\binom{D}{k} = \frac{\Gamma(1+D)}{\Gamma(1+k)\Gamma(1+D-k)}$.

The model (2) is specified by $SARFIMA(p,d,q)x(P,D,Q)_S$. When $d = 0$ and $D = 0$, the model is reduced to a classical seasonal SARFIMA model. If conditions $0 < d < 0.5$ and $0 < D < 0.5$ are satisfied the process becomes stationary.

If the process in (1) is driven by GARCH errors instead of zero white noise the process could be defined as:

$$\phi(B)\nabla^d(Y_t - \mu) = \theta(B)Z_t, \tag{7}$$

with GARCH(p,q) errors to model the conditional heteroscedasticity. That is, $Z_t|F_{t-1} \sim N(0, h_t^2)$, where F_{t-1} is the history of the process and h_t^2 satisfies the GARCH(r,s) process given by

$$h_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j Z_{t-j}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \tag{8}$$

The original process generated by (1) and could now be introduced with GARCH(r,s) errors as a $ARFIMA(p, d, q)$-$GARCH(r, s)$ process created as a result of generalized fractional differencing coupled with conditional heteroskedasticity.

Similar to transforming model (1) into model (7), model (2) could be modified by changing white noise into GARCH errors as:

$$\phi(B)\psi(B^S)\nabla^d\nabla_S^D(Y_t - \mu) = \theta(B)\Theta(B)^S Z_t \tag{9}$$

It will be a $SARFIMA(p, d, D, q)$-$GARCH(r, s)$ process created as a result of generalized fractional differencing of a SARFIMA time series coupled with conditional heteroskedasticity.

3. Methodology

In the past, parameter estimation of long memory time series models presented in the preceding section was done using state space modelling coupled with the Kalman Filter.

The state space model is comprised of a system of two equations known as the '*measurement(observation)equation*', and the '*transient(state)equation*'. The two equations could be constructed for any linear dynamic system created through the processes introduced in the previous section driven by Gaussian errors.

The resulting output of the state space configuration was thereafter filtered through a recursive algorithm known as the Kalman filter to arrive at parameter estimates of a Monte Carlo experiment. It was done for varying parameter initial values, series lengths and iterations.

Technical details of the state-spaced modelling-based Monte Carlo exercise are not discussed in this section, since they are already available for reference in the current existing body of knowledge. Especially Hagiwara [28] presents the theoretical concepts blended with practical applications of traditional state space modelling and the work has been logically dissected in an evaluation done in Dissanayake [29]. Furthermore, Hartl and Jucknewitz [30] propose a new approximation in terms of state space modelling of unobserved fractional components. Therefore it is evident that a comparative assessment of long memory models evaluated utilizing the classical maximum likelihood functions as opposed to state space modelling lacks depth in the literature.

In such a context, traditional maximum likelihood functions were employed to generate Monte Carlo estimates based on simulated data for the time series models presented in the preceding section. For the stationary, zero-mean ARFIMA model driven by Gaussian white noise the log-likelihood function is given by

$$L(\theta) = -\frac{1}{2}\log\det(\Gamma_\theta) - \frac{1}{2}y'\Gamma_\theta^{-1}y \tag{10}$$

The ARFIMA (0,d,0) maximum likelihood estimates of the parameters were obtained by maximizing Equation (10) in which $y = (y_1, y_2, y_3, \ldots, y_n)$ and Γ_θ is the variance-covariance matrix of $\{Y_t\}_{t\in Z}$ and θ being the parameter vector.

The stationary, zero-mean, Gaussian white noise-driven SARFIMA log-likelihood function is similar to (10) with a different variant-covariant matrix.

Maximum likelihood estimates for parameters λ of an ARFIMA-GARCH time series model were obtained by maximizing the conditional log-likelihood l_t of the form:

$$L(\lambda) = \frac{1}{n}\sum_{t=1}^{n} l_t l_t = -\frac{1}{2}\ln h_t - \frac{\epsilon_t^2}{2h_t} \quad (11)$$

Where $\lambda = (\gamma^T, \delta^T)^T, \gamma = (\psi_1, \psi_2, \ldots, \psi_p, \theta_1, \theta_2, \ldots, \theta_q, d)^T$ and $\delta = (\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_r, \beta_1, \beta_2, \ldots, \beta_s)^T$.

Maximum likelihood estimates of a SARFIMA-GARCH time series model were obtained by maximizing a conditional log likelihood function similar to (11) with a different set of parameters.

The simulation results of the Monte Carlo experiment that comes under the purview of this paper using Equations (10) and (11) as well as their close variants are presented in the next section. It is followed by the derivation of an estimate for the asymptotic variance of the long memory parameter in terms of the models being assessed within the scope of this article. The derivation also becomes a segment of the next section.

4. Results – monte carlo evidence

In order to evaluate the performance of the maximum likelihood method in estimating the parameters of the model, a number of Monte Carlo experiments were carried out. The simulation results provided for non-seasonally and seasonally differenced parameter estimations, the corresponding standard errors (SE), and mean square errors (MSE) of the parameters. It was carried out based on 1000 replications with different sizes of samples ($n = 100$, $n = 200$, $n = 500$ and $n = 1000$). Seasonal length was considered as 52 weeks corresponding to weekly rainfall per year. Monte Carlo results provided in this section based on rainfall data in Sri Lanka extend and corroborate the work of Silva et al. [31] and Silva et al. [32].

Results from Monte Carlo experiments are given below. For convenience and less complexity ARFIMA(0,*d*,0), ARFIMA(0,*d*,0)-GARCH(1,1), SARFIMA(0,*d*,0)x(0,*D*,0), and SARFIMA(0,*d*,0)x(0,*D*,0)-GARCH(1,1) models were chosen for the experiments. The irregular random variation noise innovations of the ARFIMA model illustrated in Table 1 were changed from Gaussian white noise to GARCH errors that lead towards the simulation results of Tables 2–5 below.

Initially, by inspecting the standard error values of d in Tables 1–13 below, the smallest (0.0244) and largest (0.10273) estimates are chosen for the analysis to arrive at an estimate for the asymptotic variance of the long memory parameter d. Thereafter, corresponding variances (0.000595 and 0.010553) were calculated. It is followed by finding the interval comprising lower and upper bounds in which the largest and smallest calculated asymptotic variance values would lie as factors in terms of corresponding series length (n) and π^2. It is an interesting exposition in standard long memory time series research due to the incorporation of a seasonal filter in some models presented in this article. Furthermore, certain models within the scope of this article were driven by white noise and others by GARCH errors. Therefore the derived asymptotic variance interval estimate of d represents

Table 1. Result for exact maximum likelihood estimator of d for a generating process of ARFIMA$(0, d, 0)$.

d	N	$\hat{d}$	SE$(\hat{d})$	MSE$(\hat{d})$
0.1	100	0.05175	0.09127	0.01066
	200	0.07485	0.06268	0.00456
	500	0.08856	0.03679	0.00148
	1000	0.09499	0.02546	0.00067
0.15	100	0.10487	0.09158	0.01042
	200	0.12658	0.05936	0.00407
	500	0.14084	0.03680	0.00144
	1000	0.14560	0.02541	0.00067
0.3	100	0.24931	0.08773	0.01027
	200	0.27264	0.05750	0.00405
	500	0.28922	0.03624	0.00143
	1000	0.29474	0.02518	0.00067
0.45	100	0.37742	0.06959	0.01011
	200	0.40795	0.04772	0.00405
	500	0.43103	0.03142	0.00135
	1000	0.44359	0.02707	0.00077

Table 2. The MLE of d, α_0, α_1 and β_1 of a generating process of ARFIMA (0,d,0)-GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.1$.

n	100	200	500	1000
$\hat{d}$	0.07505	0.08082	0.08952	0.09515
SE$(\hat{d})$	0.07783	0.06094	0.04054	0.02700
MSE$(\hat{d})$	0.00668	0.00408	0.00175	0.00075
$\hat{\alpha}_0$	0.07094	0.10044	0.13805	0.15129
SE$(\hat{\alpha}_0)$	0.11041	0.11266	0.08602	0.05281
MSE$(\hat{\alpha}_0)$	0.01844	0.01515	0.00754	0.00279
$\hat{\alpha}_1$	0.11152	0.14204	0.17934	0.19368
SE$(\hat{\alpha}_1)$	0.15044	0.13060	0.08878	0.05363
MSE$(\hat{\alpha}_1)$	0.03046	0.02041	0.00831	0.00292
$\hat{\beta}_1$	0.80155	0.72727	0.63664	0.60262
SE$(\hat{\beta}_1)$	0.25925	0.25851	0.18672	0.11003
MSE$(\hat{\beta}_1)$	0.10783	0.08303	0.03621	0.01211

Table 3. The MLE of d, α_0, α_1 and β_1 of a generating process of ARFIMA (0,d,0) – GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.15$.

n	100	200	500	1000
$\hat{d}$	0.11464	0.12679	0.13967	0.14532
SE$(\hat{d})$	0.08943	0.06719	0.04076	0.02699
MSE$(\hat{d})$	0.00925	0.00505	0.00177	0.00075
$\hat{\alpha}_0$	0.07373	0.10263	0.13947	0.15182
SE$(\hat{\alpha}_0)$	0.11536	0.11276	0.08426	0.05165
MSE$(\hat{\alpha}_0)$	0.01913	0.01496	0.00721	0.00267
$\hat{\alpha}_1$	0.11405	0.14460	0.18109	0.19436
SE$(\hat{\alpha}_1)$	0.15295	0.13012	0.08719	0.05214
MSE$(\hat{\alpha}_1)$	0.03078	0.02000	0.00796	0.00275
$\hat{\beta}_1$	0.79587	0.72149	0.63318	0.60122
SE$(\hat{\beta}_1)$	0.26576	0.25888	0.18252	0.10656
MSE$(\hat{\beta}_1)$	0.10899	0.08178	0.11619	0.12755

all such models. Therefore by inspecting the standard deviation of the estimated standard long memory parameter (d) in Tables 1–13, it is clearly evident that the asymptotic variance for all the estimates of d fall within the interval $\left(\frac{\pi^2}{17n}, \frac{\pi^2}{9n}\right)$, where n is equal to the

Table 4. The MLE of d, α_0, α_1 and β_1 of a generating process of ARFIMA (0,d,0) – GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.3$.

n	100	200	500	1000
$\hat{d}$	0.26080	0.27842	0.29125	0.29612
SE($\hat{d}$)	0.10273	0.06993	0.04059	0.02709
MSE($\hat{d}$)	0.01209	0.00536	0.00172	0.00075
$\hat{\alpha}_0$	0.07279	0.10395	0.14538	0.15580
SE($\hat{\alpha}_0$)	0.11401	0.11185	0.08353	0.04915
MSE($\hat{\alpha}_0$)	0.01896	0.01463	0.00700	0.00245
$\hat{\alpha}_1$	0.11525	0.14725	0.18548	0.19766
SE($\hat{\alpha}_1$)	0.15669	0.12942	0.08246	0.04560
MSE($\hat{\alpha}_1$)	0.03173	0.01953	0.00701	0.00209
$\hat{\beta}_1$	0.79651	0.71849	0.62025	0.59233
SE($\hat{\beta}_1$)	0.26847	0.25394	0.17527	0.09419
MSE($\hat{\beta}_1$)	0.11069	0.07853	0.03113	0.00893

Table 5. The MLE of d, α_0, α_1 and β_1 of a generating process of ARFIMA (0,d,0) – GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.45$.

n	100	200	500	1000
$\hat{d}$	0.40720	0.42797	0.44289	0.44770
SE($\hat{d}$)	0.08894	0.05994	0.03755	0.02646
MSE($\hat{d}$)	0.00974	0.00408	0.00146	0.00071
$\hat{\alpha}_0$	0.07601	0.12041	0.14941	0.15630
SE($\hat{\alpha}_0$)	0.11293	0.11916	0.08161	0.04862
MSE($\hat{\alpha}_0$)	0.01823	0.01507	0.00666	0.00240
$\hat{\alpha}_1$	0.12065	0.15949	0.18873	0.19784
SE($\hat{\alpha}_1$)	0.15684	0.12548	0.07863	0.04476
MSE($\hat{\alpha}_1$)	0.03090	0.01739	0.00631	0.00201
$\hat{\beta}_1$	0.78589	0.68169	0.61120	0.59154
SE($\hat{\beta}_1$)	0.26907	0.25938	0.16700	0.09219
MSE($\hat{\beta}_1$)	0.10696	0.07395	0.02835	0.00857

Table 6. MLE of d and D of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)_{52}$ with $d = 0.1$ and $D = 0.45$.

n	$\hat{d}$	SD($\hat{d}$)	MSE($\hat{d}$)	$\hat{D}$	SD($\hat{D}$)	MSE($\hat{D}$)
100	0.0333	0.0826	0.0112	0.4423	0.0160	0.0003
200	0.0674	0.0590	0.0045	0.4462	0.0112	0.0001
500	0.0860	0.0359	0.0014	0.4475	0.0091	0.00008
1000	0.0927	0.0297	0.0009	0.4503	0.0118	0.0001

length of the simulated time series ($n = 1000$ for the lower bound and $n = 100$ for the upper bound of interval estimate). The given interval estimate for the asymptotic variance of d is for a family of standard long memory time series, and differs from the concept of a point estimate provided for the asymptotic variance of d in a specific generalized long memory model given in Dissanayake et al. [20]. It is yet another novel contribution of this article.

Table 7. MLE of d and D of a generating process of $SARFIMA(0,d,0)x(0,D,0)_{52}$ with $d = 0.15$ and $D = 0.45$.

n	$\hat{d}$	SD($\hat{d}$)	MSE($\hat{d}$)	$\hat{D}$	SD($\hat{D}$)	MSE($\hat{D}$)
100	0.0671	0.0835	0.0138	0.4502	0.0136	0.0001
200	0.1033	0.0570	0.0054	0.4539	0.0087	0.00009
500	0.1358	0.0357	0.0014	0.4530	0.0076	0.00006
1000	0.1429	0.0260	0.0007	0.4516	0.0070	0.00005

Table 8. MLE of d and D of a generating process of $SARFIMA(0,d,0)x(0,D,0)_{52}$ with $d = 0.3$ and $D = 0.3$.

n	$\hat{d}$	SD($\hat{d}$)	MSE($\hat{d}$)	$\hat{D}$	SD($\hat{D}$)	MSE($\hat{D}$)
100	0.2236	0.0831	0.0127	0.2738	0.0690	0.0127
200	0.2607	0.0586	0.0049	0.2890	0.0386	0.0016
500	0.2846	0.0360	0.0015	0.2947	0.0260	0.0007
1000	0.2907	0.0262	0.0007	0.2978	0.0190	0.0003

Table 9. MLE of d and D of a generating process of $SARFIMA(0,d,0)x(0,D,0)_{52}$ with $d = 0.45$ and $D = 0.1$.

n	$\hat{d}$	SD($\hat{d}$)	MSE($\hat{d}$)	$\hat{D}$	SD($\hat{D}$)	MSE($\hat{D}$)
100	0.3697	0.0733	0.0118	0.0205	0.1414	0.0263
200	0.4015	0.0493	0.0047	0.0723	0.0667	0.0052
500	0.4283	0.0312	0.0014	0.0902	0.0365	0.0014
1000	0.4391	0.0244	0.0007	0.0936	0.0268	0.0007

Table 10. The MLE of D, d, α_0, α_1 and β_1 of a generating process of $SARFIMA(0,d,0)x(0,D,0)$-GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.1$ and $D = 0.45$.

n	100	200	500	1000
$\hat{d}$	0.01467	0.05484	0.0843	0.09292
SE($\hat{d}$)	0.08838	0.06595	0.04484	0.03621
MSE($\hat{d}$)	0.01509	0.00639	0.00226	0.00136
$\hat{D}$	0.45352	0.45605	0.45416	0.45195
SE($\hat{D}$)	0.01822	0.01225	0.00983	0.00946
MSE($\hat{D}$)	0.00034	0.00019	0.00011	0.00009
$\hat{\alpha}_0$	0.18195	0.22167	0.1819	0.16081
SE($\hat{\alpha}_0$)	0.18514	0.16380	0.09834	0.05415
MSE($\hat{\alpha}_0$)	0.03530	0.03197	0.01069	0.00305
$\hat{\alpha_1}$	0.07455	0.12917	0.16441	0.17743
SE($\hat{\alpha_1}$)	0.10350	0.08670	0.06012	0.04205
MSE($\hat{\alpha_1}$)	0.02645	0.01253	0.00489	0.00228
$\hat{\beta}_1$	0.68152	0.56483	0.58804	0.60484
SE($\hat{\beta}_1$)	0.29227	0.26419	0.16632	0.09476
MSE($\hat{\beta}_1$)	0.09207	0.07103	0.0278	0.00900

5. Discussion

From the simulation results provided in Tables 1–13 for similar long memory parameter values of ARFIMA(0,d,0), ARFIMA(0,d,0)-GARCH(1,1), SARFIMA(0,d,0)x(0,D,0), and SARFIMA(0,d,0)x(0,D,0)-GARCH(1,1) models the ones driven by white noise capture long range dependence better than the heteroskedastic series with GARCH errors. It is evident from the fact that ARFIMA(0,d,0) and SARFIMA(0,d,0)x(0,D,0) models return

Table 11. The MLE of D, d, α_0, α_1 and β_1 of a generating process of $SARFIMA(0,d,0)x(0,D,0)$-GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.15$ and $D = 0.45$.

n	100	200	500	1000
$\hat{d}$	0.06048	0.10293	0.13361	0.14238
SE($\hat{d}$)	0.08897	0.06597	0.04649	0.03320
MSE($\hat{d}$)	0.01593	0.00657	0.00243	0.00116
$\hat{D}$	0.45308	0.45579	0.45375	0.45202
SE($\hat{D}$)	0.01843	0.01234	0.00988	0.00914
MSE($\hat{D}$)	0.00035	0.00019	0.00011	0.00008
$\hat{\alpha}_0$	0.18121	0.22116	0.18212	0.16279
SE($\hat{\alpha}_0$)	0.18684	0.16349	0.09808	0.05617
MSE($\hat{\alpha}_0$)	0.03588	0.03179	0.01065	0.00332
$\hat{\alpha_1}$	0.07384	0.12846	0.16441	0.17651
SE($\hat{\alpha_1}$)	0.10309	0.08673	0.06031	0.04232
MSE($\hat{\alpha_1}$)	0.02654	0.01264	0.0049	0.00234
$\hat{\beta}_1$	0.6848	0.56624	0.5878	0.60321
SE($\hat{\beta}_1$)	0.29133	0.26447	0.16619	0.09872
MSE($\hat{\beta}_1$)	0.09206	0.07109	0.02777	0.00976

Table 12. The MLE of D, d, α_0, α_1 and β_1 of a generating process of $SARFIMA(0,d,0)x(0,D,0)$-GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.3$ and $D = 0.3$.

n	100	200	500	1000
$\hat{d}$	0.22345	0.25789	0.28404	0.29191
SE($\hat{d}$)	0.09362	0.06618	0.04542	0.03169
MSE($\hat{d}$)	0.01463	0.00615	0.00232	0.00107
$\hat{D}$	0.27587	0.29134	0.29554	0.29794
SE($\hat{D}$)	0.07678	0.04156	0.02733	0.01963
MSE($\hat{D}$)	0.00648	0.0018	0.00077	0.00039
$\hat{\alpha}_0$	0.1966	0.20161	0.17253	0.16032
SE($\hat{\alpha}_0$)	0.18468	0.14769	0.08641	0.05143
MSE($\hat{\alpha}_0$)	0.03628	0.02448	0.00798	0.00275
$\hat{\alpha_1}$	0.12848	0.15819	0.18011	0.18738
SE($\hat{\alpha_1}$)	0.12329	0.09166	0.06233	0.04201
MSE($\hat{\alpha_1}$)	0.02032	0.01015	0.00428	0.00192
$\hat{\beta}_1$	0.60489	0.56268	0.58478	0.59574
SE($\hat{\beta}_1$)	0.31005	0.24688	0.15138	0.09276
MSE($\hat{\beta}_1$)	0.09615	0.06234	0.02315	0.00862

smaller standard errors for the long memory parameter d on 63 of the 64 simulation results. Furthermore, the SARFIMA(0,d,0)x(0,D,0) model captures seasonality better than the ARFIMA(0,d,0) series as per the results of Tables 1–13. The results linked with the long memory parameter in Tables 1–13 provides adequate information to arrive at an interval estimate for the asymptotic variance of d across all considered standard long memory models within the scope of the study.

6. Conclusion

Based on the discussion in the preceding section it can be clearly established that the SARFIMA(0,d,0)x(0,D,0) series is the best model in terms of capturing both the features of seasonality and long-range dependence through Monte Carlo experiments run with simulated data. Comparing the results inclusive of the asymptotic variance interval estimate

Table 13. The MLE of D, d, α_0, α_1 and β_1 of a generating process of $SARFIMA(0, d, 0)x(0, D, 0)$-GARCH(1,1) with $\alpha_0 = 0.15, \alpha_1 = 0.2, \beta_1 = 0.6$ and $d = 0.45$ and $D = 0.1$.

n	100	200	500	1000
$\hat{d}$	0.36677	0.39857	0.426	0.43861
SE($\hat{d}$)	0.08139	0.05613	0.03725	0.02884
MSE($\hat{d}$)	0.01356	0.0058	0.00196	0.00096
$\hat{D}$	0.02071	0.07194	0.09022	0.09427
SE($\hat{D}$)	0.14222	0.06721	0.03566	0.02765
MSE($\hat{D}$)	0.02652	0.0053	0.00137	0.0008
$\hat{\alpha}_0$	0.18613	0.18885	0.16614	0.15776
SE($\hat{\alpha}_0$)	0.15882	0.13244	0.07836	0.04814
MSE($\hat{\alpha}_0$)	0.02653	0.01905	0.0064	0.00238
$\hat{\alpha_1}$	0.17649	0.1903	0.19467	0.19621
SE($\hat{\alpha_1}$)	0.13421	0.09716	0.06178	0.042
MSE($\hat{\alpha_1}$)	0.01857	0.00953	0.00384	0.00178
$\hat{\beta}_1$	0.56087	0.54565	0.57909	0.59065
SE($\hat{\beta}_1$)	0.29032	0.23251	0.1397	0.0877
MSE($\hat{\beta}_1$)	0.08582	0.05701	0.01995	0.00778

of the standard long memory parameter with other similar time series metrics will be a worthwhile research endeavour for the future.

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References

[1] Box GEP, Jenkins GM. Time series analysis forecasting and control. Merrifield, Va: 1976. Rev. ed. Holden-Day.
[2] Granger CWJ, Joyeux R. An introduction to long-memory time series models and fractional differencing. J Time Ser Anal. 1980;1:15–29. doi: 10.1111/j.1467-9892.1980.tb00297.x
[3] Hosking JRM. Fractional differencing. Biometrika. 1981;68:165–176. doi: 10.1093/biomet/68.1.165
[4] Geweke J, Hudak SP. The estimation and application of long memory time series models. J Time Ser Anal. 1983;4:221–238. doi: 10.1111/j.1467-9892.1983.tb00371.x
[5] Fox R, Taqqu MS. Large sample properties of parameter estimates for strongly dependent stationary Gaussian time series. Ann Stat. 1986;14(2):517–532. doi: 10.1214/aos/1176349936
[6] Dahlhaus R. Efficient parameter estimation for self-similar processes. Ann Stat. 1989;17(4):1749–1766. doi: 10.1214/aos/1176347393
[7] Sowell F. Maximum likelihood estimation of stationary univariate fractionally integrated time series models. J Econ. 1992;53:165–188. doi: 10.1016/0304-4076(92)90084-5
[8] Chen G, Abraham B, Peiris S. Lag window estimation of the degree of differencing in fractionally integrated time series models. J Time Ser Anal. 1994;15(5):473–487. doi: 10.1111/j.1467-9892.1994.tb00205.x
[9] Robinson PM. Gaussian semi parametric estimation of long range dependence. Ann Stat. 1995;23:1630–1661.

[10] Cheung Y, Diebold FX. On maximum likelihood estimation of the differencing parameter of fractionally integrated noise with unknown mean. J Econ. 1994;62:301–316. doi: 10.1016/0304-4076(94)90026-4
[11] Hauser MA. Maximum likelihood estimators for ARMA and ARFIMA models: a Monte Carlo study. J Stat Plan Inference. 1999;80:229–255. doi: 10.1016/S0378-3758(98)00252-3
[12] Wang W, Gelder PHAJMV, Vriling JK, et al. Detecting long-memory: Monte Carlo simulations and application to daily stream flow processes. Hydrol Earth Syst Sci. 2007;11:851–862. doi: 10.5194/hess-11-851-2007
[13] Chan NH, Palma W. Estimation of long-memory time series models. Adv Econ. 2006;20: 89–121.
[14] Palma W. Long memory time series theory and methods. New Jersey: John Wiley and Sons; 2007.
[15] Beran J, Feng Y, Ghosh S, et al. Long memory processes probabilistic properties and statistic properties and statistical methods. Berlin/Heidelberg: Springer; 2013.
[16] Dissanayake GS. Rapid optimal lag order detection and parameter estimation of standard long memory time series. In: Causal inference in econometrics. Cham: Springer; 2016. p. 17–28.
[17] Gray HL, Zhang N, Woodward WA. On generalized fractional process. J Time Ser Anal. 1989;10(3):233–257. doi: 10.1111/j.1467-9892.1989.tb00026.x
[18] Chung CF. A generalized fractionally integrated autoregressive moving-average process. J Time Ser Anal. 1996;17(2):111–140. doi: 10.1111/j.1467-9892.1996.tb00268.x
[19] Dissanayake GS, Peiris S. Generalized fractional processes with conditional heteroskedasticity. Sri Lankan J Appl Stat. 2012;12(0):1–12. doi: 10.4038/sljastats.v12i0.4964
[20] Dissanayake GS, Peiris MS, Proietti T. State space modeling of gegenbauer processes with long memory. Comput Stat Data Anal. 2016;100:115–130. doi: 10.1016/j.csda.2014.09.014
[21] Dissanayake GS, Peiris MS, Proietti T, et al. Estimation of generalized fractionally differenced processes with conditionally heteroskedastic errors. In: International Work Conference on Time Series. Proceedings ITISE, Granada; 2014. p. 871–890.
[22] Dissanayake GS. Advancement of fractionally differenced gegenbauer processes with long memory. Bull Aust Math Soc. 2016b;94(1):173–174. doi: 10.1017/S0004972716000241
[23] Dissanayake GS, Peiris MS, Proietti T. Fractionally differenced gegenbauer processes with long memory: a review. Stat Sci. 2018;33(3):413–426. doi: 10.1214/18-STS649
[24] Guégan D. How can we define the concept of long memory? An econometric survey. Econ Rev. 2005;24(2):113–149. doi: 10.1081/ETC-200067887
[25] Porter-Hudak S. An application of the seasonal fractionally differenced model to the monetary aggregates. J Am Stat Assoc. 1990;85(410):338–344. doi: 10.1080/01621459.1990.10476206
[26] Yaya OS, Fashae OA. Seasonal fractional integrated time series models for rainfall data in Nigeria. J Theor Appl Climatol. 2014;120(1-2):99–108. doi: 10.1007/s00704-014-1153-8
[27] Bary NK. A treatise on trigonometric series. San Francisco: Pergamon Press; 1964.
[28] Hagiwara J. Time series analysis for the state-space model with R/Stan. Singapore: Springer; 2021.
[29] Dissanayake GS. Time-series analysis for the state-space model with R/Stan. J R Stat Soc Ser A: Stat Soc. 2023;186(1):164–165. doi: 10.1093/jrsssa/qnad002
[30] Hartl T, Jucknewitz R. Approximate state space modelling of unobserved fractional components. Econ Rev. 2022;41(1):75–98. doi: 10.1080/07474938.2020.1841444
[31] Silva HPTN, Dissanayake GS, Peiris TSG. Modeling persistent and periodic weekly rainfall in an environment of an emerging Sri Lankan economy. In: Mai C, editor. Structural changes and their econometric modeling 12. Cham: Springer International Publishing; 2019. p. 314–328.
[32] Silva HPTN, Dissanayake GS, Peiris TSG. The use of fractionally autoregressive integrated moving average for the rainfall forecasting. In: Beyond traditional probabilistic methods in economics 2. Cham, Ho Chi Minh City: Springer International Publishing; 2019. p. 567–580.