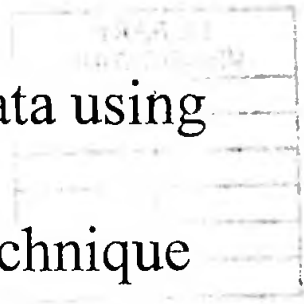


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Interpretation of Potential field data using Backus and Gilbert Inversion Technique



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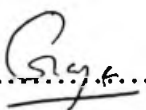
Backus and Gilbert Inversion Technique

by

Yapa Arachchilage Gayani Sumunduni Yapa

Thesis submitted to the University of Sri Jayewardenepura for the
award of the Degree of Master of Philosophy in Mathematical
Physics on 2013.

“The work described in this thesis was carried out by me under the supervision of Professor D.A.Tantrigoda and Professor Arjuna de Zoysa and a report on this has not been submitted in whole or in part to any University or any other institution for another Degree/Diploma”.

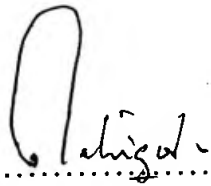
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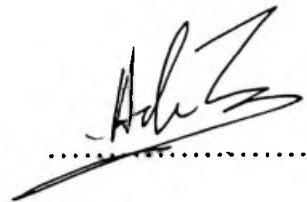
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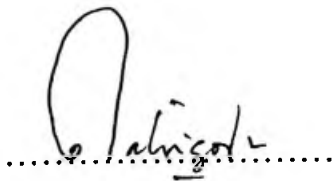


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I certify that the candidate has incorporated all corrections, additions and amendments recommended by the examiners.

A handwritten signature in black ink, appearing to read "D.A. Tantrigoda", is written over a horizontal dotted line. The signature is cursive and somewhat stylized.

Prof. D.A. Tantrigoda

Internal Supervisor

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Acknowledgements

I take this opportunity to express my heartfelt gratitude and deep regards to my supervisor and mentor Professor D.A. Tantrigoda, Chair and Senior Professor of Physics, University of Sri Jayewardenepura for his perfect guidance, supervision and constant encouragement throughout the Project. The blessings help and guidance given by him when needed shall indeed carry me a long way in my future endeavors.

I would also like to thank Professor Arjuna de Zoysa for the assistance and encouragement given and also take this opportunity to express a deep sense of gratitude to Mr. Dileepa Witharana, Head of the department of mathematics and philosophy of Engineering, the Open University of Sri Lanka for his cordial support, valuable information and guidance, which helped me in completing this task through various stages.

I am obliged to staff members of University of Sri Jayewardenepura, for the valuable information provided by them in their respective fields. I am grateful for their cooperation during the period of my assignment.

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ABSTRACT

The main objective of this study is to test the hypothesis that gravity anomalies could be modeled in terms of bodies having density that vary with depth using the Backus and Gilbert inversion method. The hypothesis was tested using a series of numerical experiments involving calculation of the gravity anomaly due to model geological structures of known dimensions and known vertical density distributions and then modeling them using the Backus and Gilbert method. This in fact is a problem in the linear inverse theory and such problems are ill posed in general. To overcome this difficulty the Singular Value Decomposition method has been successfully adopted.

The study was conducted with simulated data and to make them resemble real observations, a certain amount of random errors were added. To recover the assumed structure of the causative body, "weighted distance" minimization was applied at the latter stages of the inversion process. The method presented in this study can be used successfully to model gravity anomalies caused by;

- (1) Common geological structures (sedimentary basins, igneous intrusions and geological faults) with densities that vary with depth closest to a given relationship and

(2) A sedimentary basin of constant density having a high dense igneous layer intruded into it. Density variations with depth have been expressed in terms of local averages using averaging kernels.

This method may have useful applications in the oil and gas exploration work especially in view of its ability to identify thin igneous layers intruded in to sedimentarybasins.