A self-tuning modified firefly algorithm to solve univariate nonlinear equations with complex roots

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Abstract—**The use of numerical methods to solve univariate nonlinear equations has many drawbacks. We propose a modified firefly algorithm [MOD FA] with a self-tuning ability to solve a given univariate nonlinear equation. Our modification is capable of finding almost all real as well as complex roots of a nonlinear equation within a reasonable interval/range. The modification includes an archive to collect best fireflies and a flag to determine poorly performed iterations. It is also capable of tuning the algorithm-specific parameters while finding the optimum solutions. The self-tuning concept allows the users of our application to use it without any prior knowledge of the algorithm. We validate our approach on examples of some special univariate nonlinear equations with real as well as complex roots. We have also conducted a statistical test: the Wilcockson sign rank test. By conducting a comparison with the genetic algorithm and differential evolution with same modifications [MOD GA] [MOD DE] and with the original firefly algorithm [FA], we confirm the efficiency and the accuracy of our approach.**

I. Introduction

Nonlinear equations play a vital role in most of the real world applications. Many fields, including engineering, mathematics, chemistry, computer science and economics often require applications of univariate as well as systems of nonlinear equations. Solving a univariate nonlinear equation $f(x) = 0$ is finding roots α, such that $f(\alpha) = 0$. Providing solutions for such nonlinear equations is challenging and the common method of solving them is the use of numerical methods; specially the techniques based on Newton's method [1], [2], [3], [4]. Numerical methods often have requirements to be fulfilled to begin with the process of finding approximations. These requirements can be considered as drawbacks of numerical methods that users encounter when using them to solve nonlinear equations. Need of the derivative information and the continuity of the function, evaluation of large matrices, inability to give more than one approximation at a time, depending over the sensitivity of the initial guess and the slow convergence are some of the major drawbacks. Thus, finding better approaches to solve nonlinear equations are still open for research.

In practice, most nonlinear equations are solved using Newton's method or its variants. Weerakoon & Fernando have suggested an improvement to the existing Newton's method in their paper: A Variant of Newton's Method with Accelerated Third-Order Convergence [1]. The method involves changing the derivation of Newton's method. The derivation of the Newton's method involves approximating an indefinite integral of the derivative of the function by a rectangle. Weerakoon & Fernando have modified it to be a trapezium so that the error of the approximation is reduced. The researchers proved that the order of convergence of the suggested modification is three. In fact, for some cases it is even higher than three. The main concern in this research was on the speed of convergence to the approximation. But this method also contains the aforementioned drawbacks of numerical methods such as the need for derivative calculation.

Apart from the numerical methods, heuristics were also being proposed to solve nonlinear equations. A continuous global optimization heuristic: Continuous Greedy Randomized Adaptive Search Procedure known as C- GRASP has been adopted by Michael J. Hirsch et al to solve a given system of nonlinear equations [5]. They address the problem of finding all the roots of a system of equations assuming that all roots are real. The heuristic does not use the derivative information of the equations of the system. The attempt of the researchers was successful, but they haven't addressed the area of complex roots.

Nature inspired algorithms; being meta-heuristic for most of the time, are now becoming the dominator of the world of optimization algorithms. Compared with other methods, these algorithms have many advantages. Some examples are genetic algorithm, differential evolution, particle swarm optimization, harmony search, firefly algorithm, cuckoo search and others [6], [7], [8], [9], [10], [11]. The meta-heuristic property makes them robust so that these algorithms are capable of touching a variety of problems [12], [13], [14]. Since the applicability of these algorithms is immense, researchers have done some experiments over adopting nature inspired algorithms to solve nonlinear equations.

Use of genetic algorithm to solve nonlinear equations has been carried out in a research done by Nikos E. Mastorakis