Determination of the geoid of central highlands in Sri Lanka

by

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"The work described in this thesis was carried out by me under the supervision of Professor D.A.Tantrigoda and a report on this has not been submitted in whole or in part to any University or any other Institution for another Degree/Diploma".

Signature of the candidate

19-09-2008
Date

"I certify that the above statement made by the candidate is true and that this thesis is suitable for submission to the University for the purpose of evaluation".

Signature of supervisor

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ABSTRACT

Geoidal undulations over two hundred GPS stations have been interpolated using least-squares surface fitting technique to determine the geoid of central highlands in Sri Lanka. However, it is not possible to interpolate these points directly to prepare a detailed map of geoidal undulations as its component due to the topography is rapidly varying making the interpolation inaccurate. Gravity potential and the undulation of the local geoid due to the topography have been calculated separately using a topographic model and removed from the above available geoidal undulations. Resulting data were interpolated using a least-squares technique and the Bouguer co-geoid was determined as two different cases. In the first case, a single mathematical surface was fitted using all reduced geoidal undulations situated over the entire Sri Lanka. In the second case, three surface polynomials were employed to determine the Bouguer co-geoid for the same area. Finally, the geoidal undulations due to the topography were added back to each of the above mathematical surface to create a detailed map of the geoid of central highlands in Sri Lanka. In both cases, local positive geoidal surfaces superimposed on a large negative regional surface have been obtained and the minimum value of the geoidal undulation is about -92.05m in the vicinity of Piduruthalagala peak.

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1. Introduction

1.1. Background

The initial survey activity for a country is the establishment of its geodetic control network. Before the advent of new technology, triangulation was the only process that was available to establish control points in Sri Lanka. When Electromagnetic Distance Measurement (EDM) technology and Global Positioning System (GPS) were introduced, this became a relatively convenient task. In 1992, the revision work of the geodetic control network was commenced using GPS and in 1994 the country's first high accurate GPS network was established.

Although, the planimetric accuracy of control points provided by triangulation was improved by use of the GPS technology, the heights of triangulation stations from the Mean Sea Level (MSL) had been calculated employing the reciprocal vertical angle method. The MSL extended for the whole of the earth is generally referred to as the geoid. In view of the inaccuracy of this method, the method of precise levelling, as the procedure to determine heights between points on the Earth surface, had been undertaken since 1909 by the survey department of Sri Lanka. The primary levelling network covered the entire country and was then densified by establishing secondary, tertiary and minor levelling networks to provide height control for all the development projects in Sri Lanka.

The height measured from the MSL, usually defined as the orthometric height, is an important parameter in any survey. Many countries tend to use the GPS technology for obtaining orthometric heights as the levelling procedure is time consuming. However GPS technology can not be used directly for obtaining orthometric heights. In the GPS system, heights are measured from an ellipsoidal surface called World Geodetic System 1984 (WGS 84) which has been defined using relevant parameters. Therefore, measured ellipsoidal heights using the GPS technique require the position of the geoid relative to the ellipsoid in order to determine the corresponding orthometric heights.

As explained above, computation of orthometric heights is still being a problem unless the precise geoidal surface is known.

1.2. Problem Formulation

As indicated above, the geoid is usually determined relative to the WGS 84 reference ellipsoid. The relationship between the geoid and the reference ellipsoid is given by their linear separation called geoidal undulation (N) and the deflection of the vertical (θ), the angular deviation between normal to the ellipsoid and that of to the geoid. Since deflection of the vertical is very small and can be neglected for most practical purposes, geoidal undulation is the major factor representing the relationship between the geoid and the ellipsoid. Therefore, the relationship between the ellipsoidal height (h), the corresponding orthometric height (H) and the geoidal undulation (N) can be expressed using the simple equation,
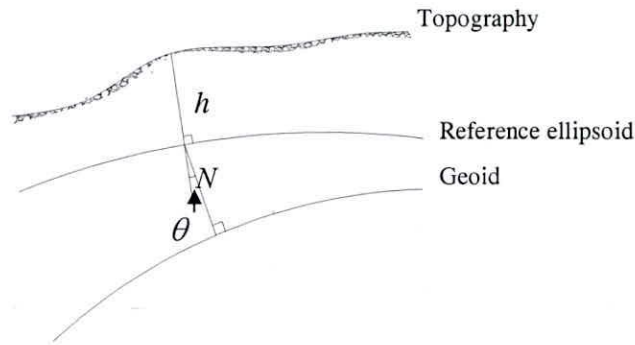
$$h = H + N \tag{1.1}$$

Figure 1-1: Geoid and reference ellipsoid

At present in Sri Lanka, geoidal undulations are available for little over 200 GPS stations and 50 trignometrical stations situated in various parts of Sri Lanka. Based on the available data, geoidal surface can be interpolated, for an area where there are no large topographical features, using the least-squares surface fitting technique. Surface polynomials of different degrees are fitted to the above geoidal undulations and the one, which gives the least residuals, is selected as the best surface polynomial that approximates to the geoid. After determining the best geoidal surface over the entire country, geoidal undulation can be obtained as a function of position on the surface. Hence the orthometric heights can directly be computed by direct substitution in the above equation.

Geoidal undulations depend on the topography as well as sub-surface mass anomalies. The component that depends on mass anomalies (Bouguer co-geoid) has smooth variations as it is being caused by deep seated sources. However, the component that depends on the topography has large variations especially in areas where there are rugged topography such as the central highlands in Sri Lanka.

Therefore, geoidal undulations can not be determined accurately by mere interpolation of available undulations mentioned above due to problem that will be caused by the component which has rapid variations. Hence, first the component that depends on the topography has been removed from the available undulations before fitting a mathematical surface. After determining the Bouguer co-geoid, the removed topographic effect of undulation has been added back to obtain the total geoidal undulation.

1.3.   Objectives of the Study

1 Determination of the mathematical surface which approximates to the geoid of Sri Lanka.

2 Computation of orthometric heights using GPS derived ellipsoidal heights with geoidal undulations.

1.4.   Thesis structure

Chapter 1 gives an introduction to outline the general context of the work, including the background of the study, the research problem solved in this research work, objectives of the study and research methodologies.

Chapter 2 deals with the presentation of the literature review on various methods of determination of the geoid. This includes gravimetric, astrogeodetic and least-squares surface fitting methods.

Chapter 3 describes the methodology of determination of the geoid of Sri Lanka including the theoretical background, the construction of various topographic models and different methods of surface fitting technique.

Chapter 4 focuses on the results obtained by the two different surface fitting techniques and the comparison of their accuracies using a model validation.

Chapter 5 gives the conclusions and recommendations for future work.

2. Review of the methods of determination of the geoid

2.1. Gravimetric geoid determination

Geoid is defined as the equipotential surface of the earth's gravity field coinciding with the mean sea levels of the oceans. Averaging the ocean surface over a time (at least over one year) or modeling ocean tides provides mean sea levels (MSL) for the corresponding time interval (Torge, 2001).

According to Newton's law of gravitation, the Earth's effective gravitational potential W at point on the surface of Earth is expressed by the formula:

$$W = V + \phi = G\iiint_v \frac{\rho dv}{r} + \frac{(l\omega)^2}{2} \tag{2.1}$$

Where V is gravitational potential of the earth;

   ϕ is centrifugal potential of the earth;

   dv is an element of unit volume inside the earth of total volume v;

   r is the distance from the mass element to a specified point;

   ω is the angular velocity of the earth; and

   l is the perpendicular distance from the axis of rotation