

**University of Sri Jayewardenepura
Faculty of Graduate Studies**

Department of Mathematics

**A MATHEMATICAL MODEL FOR
POPULATION GROWTH, BIRTH AND DEATH RATES OF SRI LANKA**

**A Thesis
By
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**Submitted in Partial Fulfillment
of the requirements
for the Degree of
Master of Science
In
Industrial Mathematics.**

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DECLARATION

I hereby declare that the M.Sc. thesis titled “ A mathematical model for population growth, birth and death rates of Sri Lanka” contains my own work and that no part of this thesis has been previously submitted to this or any other university or institution in application for admission to a degree, diploma, or any other qualification.


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Abstract

In the last few decades many people have done population projections. Most of the projections were based on certain assumptions about fertility and mortality. No attempt has been made to assess the validity of the assumptions. Our main objective is to develop a model that can be used to predict future population sizes. We check the validity of the models by examining the accuracy of the predictions made using the suggested models. Two kinds of models are developed, namely, component and noncomponent models. In the component model, birth rates and death rates are separately modeled and are used to estimate the number of births and deaths. The population figures are then adjusted for the number of births and deaths in a specified period. Finally, we develop age specific birth and death rate models to predict future birth and death rates of different age groups.

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Chapter 1

Introduction

1.1 Introduction

Essentially the word “population” can be used to describe a group of similar individuals or objects. Here we concentrate on human populations. In the normal practice, most of the countries keep track of the population size by having a periodic census. In Sri Lanka, it has been the practice of having a national census every ten years.

Having a census is not the only way for keeping track of population sizes. On one hand, taking a census is a costly exercise making it impossible to have frequent census. On the other hand, it is the knowledge of the future population that is needed in planning. Consequently, precise estimation of population sizes based on the past information is essential for any kind of population.

Population projections are essentially mathematical models which attempt to indicate what the future population would be if certain likely rates of mortality, fertility and migration were to apply. The accuracy of the projections, to a great extent, depends on the validity of the assumptions. Despite the uncertainty about their accuracy, population projections are of utmost importance in the formulation of comprehensive development plans and programmes. If the future population can be predicted with some certainty, then the government and other responsible bodies can plan ahead to retain the living conditions of the people and to improve it if an improvement is needed.

There are numerous advantages of studying population statistics. The most general facts among these include finding out what happens and formulating theories as to why it occurs and

testing these theories against the course of events. Secondly, There is a practical value in measuring population growth so as to be able to plan ahead for satisfying man's needs throughout life such as food, clothing, shelter and other necessities. Thirdly, there is the aspect of the subject, which some times called 'political arithmetic', the term that some pioneers of Demography used to indicate the nature of there interest in population.

The rate of growth of the population and its future size and composition help to determine targets of production in various fields of economic activities. The number of people who will enter the labour force and for whom avenues of productive employment have to be created has again to be estimated on the basis of likely increase and structure of the population. Plans for education have to take into consideration, the estimates of the future population of school –going age while the growth and distribution of population will govern policies regarding location and expansion of health, housing and other social amenities. In short, any type of planning for the development of a country involves directly or indirectly some assumptions regarding the size and composition of the future population.

Population projections has thus become and important factor in drawing socio-economic plans. Population changes involve three separate components: births, deaths and migration. But in most of the cases, migration is assumed to be nil. Models that consider separate effects of each of these components are known as component models. Models that directly use the net effect of the three components are referred to as non-component models. These two projection methods are extensively used in the field of population projections.

Chapter 2 discusses literature survey. In this chapter, we focus on what others have studied with regard to birth rates, death rates and population in Sri Lanka. Chapter 3 deals with modeling birth rates for Sri Lanka. Chapter 4 suggests on models for death rates in Sri Lanka. In Chapter 5, we try to test the applicability of the suggests models for United States. Chapter 6 focuses on modeling population size in Sri Lanka. Chapter 7 discusses on age specific models for birth and death rates in Sri Lanka. Chapter 8 reports on conclusions and further work. It explains how we achieve our objectives. Some of the further work that can be done is also suggested.