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A Strategy to Reduce the Waiting Time at the Outpatient Department of the National Hospital in Sri Lanka

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Abstract -The National Hospital of Sri Lanka is a major healthcare service provider in the country. The hospital has been expanded with many healthcare facilities. The out-patient department (OPD) is the main division of the hospital which provides their service to over two thousand patients daily. Queuing is a major challenge for healthcare services all over the world, particularly in the developing countries. This study investigates the application of queuing theory to reduce the patients' waiting time at the OPD of the National Hospital of Sri Lanka. In this study, consultation and pharmacy at the OPD were only considered. The secondary data were collected under the patients' arrivals and doctors' roster. The primary data were collected through direct observations during seven weekdays. Data were analyzed and used to model channels for each section of the OPD.

Index Term - OPD, patient, Queuing theory, service rate, waiting time.

I. Introduction

The National Hospital was established in 1861, with bed strength of only hundred, and now it is a premier teaching hospital, with tertiary care facilities in the country. This has expanded into a hospital with seventy five wards of bed strength over three thousand, thirty five operation theaters, twelve intensive care units, providing a wide spectrum of services to the country. The main services are Accident service, Anesthesia and Intensive care, Burns unit, **Out patients department (OPD)**, Vascular surgery Urology, and Cardiology. OPD of the National Hospital is the main service area among all the services. It serves large number of patients every day. This department has been developed over the past years. There are eight medical units with seven hundred and fifty beds and a medical intensive care unit with eight beds. There is a health promotion unit which also provides dialysis and endoscopy services and an operating theatre. Undergraduate and post graduate trainings are also provided.

The main consultation room of the OPD is room number fifteen. There are three registration counters and seventeen doctors' tables and three pharmacy counters. A large number of patients' traffic can be seen daily in the waiting area with wooden benches in front of the room 15. The OPD opens 24 hours on weekdays and from 6 a.m. to 12 noon on holidays and weekends. A large crowd can be seen from early in the morning to the noon. The OPD staff is very busy during this period with handling long waiting queues. Therefore, this study mainly focused on the patients who need the service of the consultation room. Furthermore, the patients waiting in pharmacy are also considered.

The consultation area consists of doctors' tables, patients' checking area, an attendants' changing room and a store room. The space available for pharmacy area is also small. Though there are three counters, they generally open two counters. The arrangements of the counters and the number of doctors at the OPD have no specific reason to be organized in this manner. The OPD is more crowded during 8 a.m.-12 noon and the rush is more critical around the consultation room comparatively to the pharmacy. Hence the decision was taken to focus more on the patients in the consultation than pharmacy.

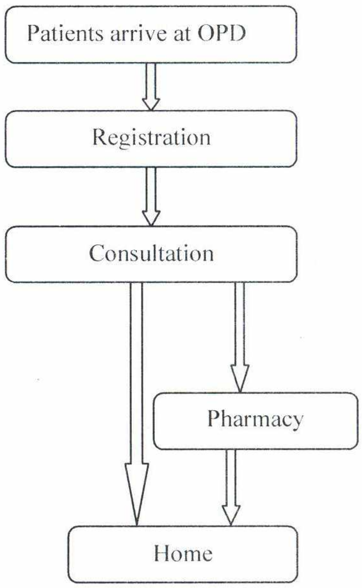

Figure 1: The flow chart of OPD

II. Materials and Methods

Queuing theory is a mathematical approach that deals with one of the most unpleasant experiences of life, is waiting. Queuing has increasingly become a common management tool for decision

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making. This vital tool is unfortunately minimally used in most healthcare systems [1]. A queuing system can be described by arrival pattern of patients, the service system, the "queue discipline", and patients' behavior [2].

Arrival characteristics

The arrivals of patients for a service system have three major characteristics: *Size of the arrival population, Behavior of arrivals, and Pattern of arrivals.*

Size of the arrival population:

There are about two thousand patients arrive daily for normal consultation of the OPD. Although there is a limited waiting room, the population size is not limited. When the number of patients or arrivals on hand at any given moment is just a small portion of all potential arrivals, the arrival population is considered unlimited, or infinite. The service of the OPD is provided until the last patient who arrives to the hospital. Most queuing models assume that an infinite arrival population.

Pattern of Arrivals at the System:

In general, patients arrive at a service facility either according to some known schedule (for example, one patient every ten minutes) or else they arrive randomly. According to this study the arrivals are random (Arrivals are considered random when they are independent of one another and their occurrence cannot be predicted exactly). The number of arrivals per unit time can be estimated by a probability distribution known as the Poisson distribution [3].

Service Characteristics

Two basic properties are important on providing service to the patients, namely, design of the service system and the distribution of service times

Service systems are usually classified in terms of their number of channels (number of servers) and number of phases (number of service stops). **They are single-channel queuing system, multi-channel queuing system, single-phase system, and multi-phase system.** The current queuing situation at the OPD can be identified as a *multi-channel queuing system* because system is with one waiting line but with several servers. The distribution of service time is assumed to be an exponential distribution.

Queue Discipline

This refers to the rule by which patients in the line are to receive the service. While in line, patients may be chosen for service by allocation to the channels in an ordered first-come-first-served (FCFS) manner or at random. Patients may be chosen for the service on a last-come-first-served (LCFS) basis. But most queuing systems use FCFS. The queue discipline of this study is assumed as FCFS although there are some priorities in the system.

Patients' Behavior

Patients' behavior can vary. Arriving patient may balk (not join the queue) because of the length of the existing queue, or simply because they do not want to wait at all, and eventually lost of getting service.

Sometimes they lost because they have no opportunity to wait. Several patients may be to collusion whereby only one person waits in line while the rest are then free to attend to other things. Some may even arrange to take turns of waiting and some may jockey from one line to another or may lose patience and leave the line [4]. The above mentioned all behaviors of patients can be seen at OPD in smaller percentages.

Types of Queuing System

There are four major types of queuing system, namely, **single-server single-phase system, single-server multi-phases system, multi-servers single-phase system, and multi-servers multi-phases system.**

Multi-servers, single-phase queuing system characterized by a situation whereby there is a more than one service facility providing identical service but drawn on a single waiting line. This is the type of queuing system practiced at the OPD of hospitals. Moreover, consultation and pharmacy are considered as separate phases. A single waiting line and several servers can be seen in each section. Since the population is unlimited, the queue is infinite, and the queuing system is multi-channel, which is called M/M/C model. Some assumptions were made in this study such as patients awaiting service form one single line and then proceed to the first available server, arrivals follow a Poisson distribution and service times are exponentially distributed, service discipline is 'FCFS', and all servers are performed at the same rate.

The following queuing parameters and formulas were used for calculation. Average arrival rate λ, average service rate at each channel μ, the number of channels c, server utilization $a = \frac{\lambda}{\mu}$ (one server), server utilization $\rho = \frac{\lambda}{\mu c}$ (multi-server), the probability of zero patients in the system $P_0 = \left(\sum_{k=0}^{k=c}\left(\frac{a^k}{k!}\right) + \left(\frac{a^c}{c!}\right)\left(\frac{1}{1-\rho}\right)\right)^{-1}$, the average number of patients waiting for service $L_q = P_0\left(\frac{a^c}{c!}\right)\frac{\rho}{(1-\rho)^2}$, the waiting time in the queue $W_q = \frac{L_q}{\lambda}$, the average service time $W_s = \frac{1}{\mu}$, the total waiting time $W = W_s + W_q$, and the average number of patients in the system $L = W\lambda$.

The current queuing system in the National Hospital is mostly inefficient ($\rho > 1$). The following table shows inefficient situation in the hospital in selected time periods in only 4 days.

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Table 1: The current server utilization for the consultation

Day	Hour	Average No of channels	Arrival rate λ/min	Service rate μ/min	ρ = λ/μc
1	8-9 a.m.	4	5.5500	0.5648	2.4566>1
	9-10 a.m.	4	4.7000	0.5648	2.0804>1
	10-11 a.m.	4	3.5500	0.5648	1.5714>1
	11-12 a.m.	3	2.2167	0.5648	1.3083>1
2	8-9 a.m.	2	4.3833	0.5648	3.8804>1
	9-10 a.m.	4	4.8000	0.5648	2.1246>1
	10-11 a.m.	9	3.2500	0.5648	0.6394<1
	11-12 a.m.	11	2.8333	0.5648	0.4560<1
3	8-9 a.m.	5	3.4667	0.5648	1.2276>1
	9-10 a.m.	7	3.1833	0.5648	0.8052<1
	10-11 a.m.	8	3.8833	0.5648	0.8594<1
	11-12 a.m.	8	2.3167	0.5648	0.5127<1
4	8-9 a.m.	6	3.7000	0.5648	1.0918>1
	9-10 a.m.	9	5.7000	0.5648	1.1213>1
	10-11 a.m.	8	5.4000	0.5648	1.1951>1
	11-12 a.m.	4	3.3667	0.5648	1.4902>1

Thus the number of channels for the service is not sufficient enough with the current arrival rates. Most of the hours in four days show the inefficient situations with the current number of channels for the service. Therefore, the existing system has to be modified to make it more efficient. For that purpose, the data were collected as follows.

The secondary data were collected of the period from January to March in 2014 during 6 a.m. to 20 p.m. for the consultation. The arrivals of patients for the consultation in each hour of the day were taken to calculate average arrivals for entire weekdays. Since the service of the OPD is only provided until 12 noon on weekends and holidays, the calculations of those days were done separately from weekdays. Since the majority of the population of the country is Buddhists, religious holiday of them was considered separately from other holiday.

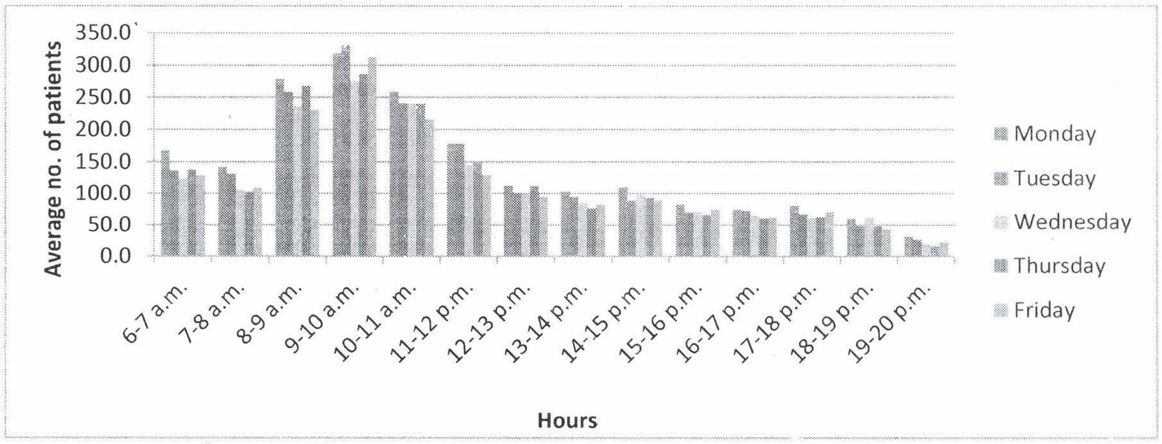

Figure 2: Average arrivals of patients for the consultation on weekdays

Table 2: Arrival rates (patients/ minute) for the consultation on weekdays

Hour	Mon	Tue	Wed	Thu	Fri
6- 7 a.m.	2.79	2.28	2.07	2.31	2.15
7-8 a.m.	2.36	2.18	1.76	1.71	1.83
8-9 a.m.	4.65	4.31	3.96	4.48	3.85
9-10 a.m.	5.30	5.51	4.59	4.78	5.22
10-11a.m.	4.32	4.02	4.00	4.01	3.62
11-12 p.m.	2.97	2.98	2.43	2.48	2.16
12-13 p.m.	1.88	1.68	1.65	1.87	1.59
13-14 p.m.	1.71	1.58	1.41	1.29	1.38
14-15 p.m.	1.84	1.49	1.63	1.56	1.50
15-16 p.m.	1.39	1.18	1.19	1.11	1.25
16-17 p.m.	1.24	1.22	1.09	1.02	1.04
17-18 p.m.	1.36	1.12	1.03	1.05	1.18
18-19 p.m.	1.01	0.82	1.01	0.81	0.73
19-20 p.m.	0.52	0.45	0.33	0.27	0.37

According to the arrivals of patient shown in the figure 2, the arrivals are skewed towards the left; it shows that the arrivals fallow Poisson distribution. The table 2 shows the average arrival rates hourly on five week days for consultation. The minimum and maximum number of arrivals can be seen around 19-20 p.m. and 9-10 a.m. respectively. When the arrivals on weekends and holidays for the consultation were considered, the following situations could be found.

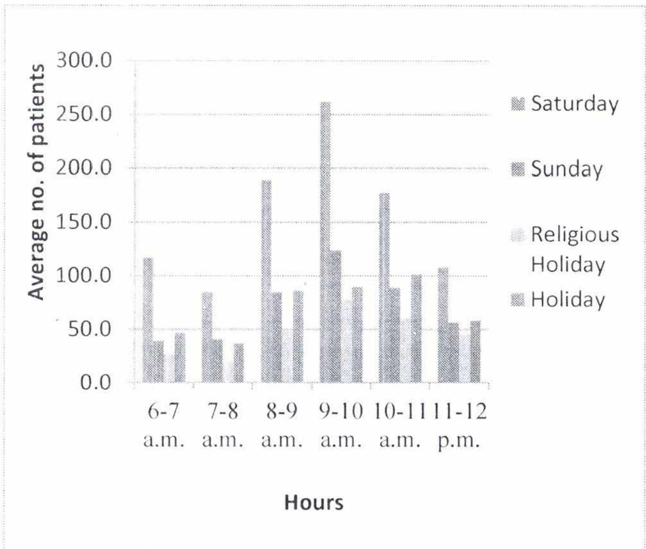

Figure 3: Average arrivals of patients for the consultation on weekends and holidays

Table 3: Arrival rates (patients/ minute) for the consultation on weekends and holidays

Hour	Sat	Sun	Religious holiday	Holiday
6-7a.m.	1.95	0.66	0.47	0.79
7-8a.m.	1.42	0.69	0.33	0.62
8-9a.m.	3.16	1.41	0.83	1.45
9-10a.m.	4.37	2.07	1.30	1.50
10-11a.m.	2.96	1.49	1.01	1.69

According to the figure 3 and table 3, arrivals of patients for the consultation on Saturday are higher than the other days and the minimum number of arrivals can be seen on religious holidays. The skewness of arrivals is same as the morning sessions on weekdays. The arrivals for consultation were considered separately for pharmacy. Therefore, the data collection and analysis for pharmacy have done as follows.

The primary data were collected through direct observations on seven weekdays during 8 a.m. to 12 noon for pharmacy since there were no records. The arrivals rates for each hour for pharmacy were calculated. The following figure 4 and table 4 show the arrivals of number of patients and its rates.

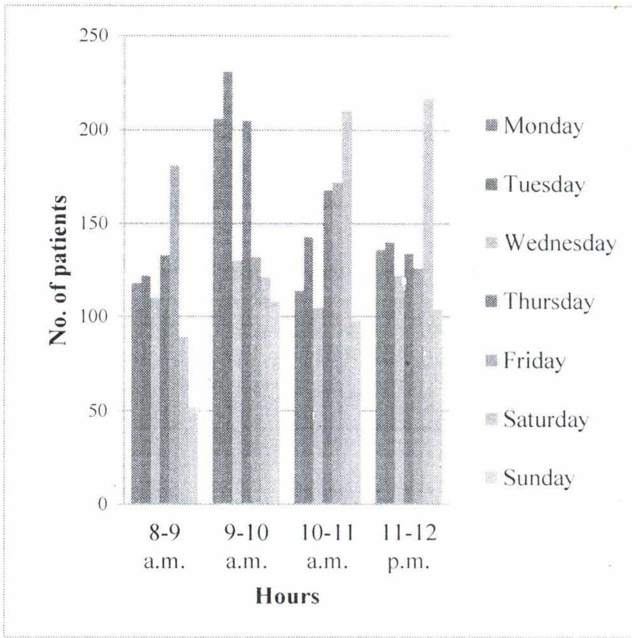

Figure 4: Arrivals of patients for the pharmacy

Table 4: Arrival rates (patients/ minute) for the pharmacy

Hour	Mon	Tue	Wed	Thu	Fri	Sat	Sun
8-9 a.m.	1.97	2.03	1.83	2.22	3.02	1.48	0.87
9-10 a.m.	3.43	3.85	2.17	3.42	2.20	2.02	1.80
10-11a.m.	1.90	2.38	1.75	2.80	2.87	3.50	1.63
11-12p.m.	2.27	2.33	2.03	2.23	2.10	3.62	1.73

According to the above figure 4 and table 4, arrivals are high during 9-12 p.m. Normally, there are three counters issuing drugs to the patients. The counters are not sufficient to fulfill the service. Patients wait for the service in line without any benches or chairs. Therefore, the queuing system of the pharmacy also should be improved by reducing waiting time and lengthy queues.

First of all, the average service rate of consultation is calculated using data on 4 days which is shown in table 1. That is **0.5648 patients/ minute.** The service rate for all channels was assumed to be same. Same analysis has done for the pharmacy. The average service rate of pharmacy counters was calculated as **1.1021 patients/ minute.**

III. RESULTS

Using M/M/C model the length of the queue, waiting time in the queue, and all other parameters were calculated by changing the number of channels for each hour. The part of the result which was obtained in the analysis is given in table 5 for early hours on Monday. When the number of channels is increased the length of the queue vanishes and waiting time decreases. According to the current number of channels for the consultation, the queuing system is inefficient. The empty columns which are in front of the occupied columns in each row show the inefficient situations and the other empty columns show non-cost effectiveness. The calculation has been stopped when the length of the queue is less than or equal to one.

Table 5: Calculation of parameters for the consultation

Hour	Parameters	No. of channels											
		1	2	3	4	5	6	7	8	9	10	11	12
6-7a.m.	p					0.9886	0.8238	0.7061					
	P_0 (%)	INEFFICIENT				0.0452	0.4934	0.6383					
	Lq (Patients)					83.9634	2.5615	0.7464		NOT COST EFFECTIVE			
	Wq (min)					30.0761	0.9498	0.2674					
	Ws (min)					1.7705	1.7705	1.7705					
	W (min)					31.8466	2.7203	2.0379					
	L					88.9063	7.5943	5.6892					
7-8a.m.	p					0.8369	0.6974						
	P_0 (%)					0.9575	1.3469						
	Lq (Patients)	INEFFICIENT				3.2181	0.7646			NOT COST EFFECTIVE			
	Wq (min)					1.3617	0.3235						
	Ws (min)					1.7705	1.7705						
	W (min)					3.1323	2.0941						
	L					7.4025	4.9489						
8-9a.m.	p									0.9151	0.8236	0.7487	
	P_0 (%)									0.0129	0.0207	0.0240	
	Lq (Patients)					INEFFICIENT				7.8506	2.1658	0.8431	
	Wq (min)									1.6877	0.4656	0.1812	
	Ws (min)									1.7705	1.7705	1.7705	
	W (min)									3.4582	2.2361	1.9518	
	L									16.0866	10.4018	9.0791	

93

The above table 5 shows the calculations of parameters on Monday from 6 a.m. to 9 a.m. hourly for the consultation. The required number of channels for the service has been increased by 8 a.m. The inefficient situations arise when the server utilization is greater than one with the available number of channels. The number of channels can be increased further. For that, this proposed system should be applied first and then a cost analysis should be done. According to these results, the following optimal result was obtained for consultation. The table 6 and 7 show the number of channels/ doctors required for the consultation when the length of the queue is less than or equal to one for each hour of the day.

Table 6: The number of channel/ doctors required for the consultation on weekdays

Hour	Mon	Tue	Wed	Thu	Fri	Average
6-7 a.m.	7	6	5	6	6	6
7-8 a.m.	6	6	5	5	5	5
8-9 a.m.	11	10	9	11	9	10
9-10 a.m.	12	13	11	11	12	12
10-11a.m.	10	10	10	10	9	10
11-12 p.m.	7	7	6	6	6	6
12-13 p.m.	5	5	4	5	4	5
13-14 p.m.	5	4	4	4	4	4
14-15 p.m.	5	4	4	4	4	4
15-16 p.m.	4	3	3	3	4	3
16-17 p.m.	4	3	3	3	3	3
17-18 p.m.	4	3	3	3	3	3
18-19 p.m.	3	3	3	3	2	3
19-20 p.m.	2	2	1	1	2	2

Table 7: The number of channels/ doctors required for the consultation on weekends and holidays

Hour	Sat	Sun	Religious holiday	Holiday
6-7 a.m.	5	2	2	2
7-8 a.m.	4	2	1	2
8-9 a.m.	8	4	3	4
9-10 a.m.	10	5	4	4
10-11 a.m.	7	4	3	5
11-12 p.m.	5	3	2	3

According to the arrivals of patients in each day the number of channels was proposed in table 6 and 7. Since there are a large number of patients in the morning, the required number of channels on five weekdays is high. Therefore, the number of channels was adjusted according to the arrivals. There may be various reasons for this arrival patterns such as transportation problems, severity of the illnesses, having thought of coming early in the morning and so on. The arrivals of patients on Saturday are higher than other holidays. Therefore, the required number of channels is high on Saturday when compared with other holidays. The same analysis was done for the pharmacy and the following table 8 and 9 show the number counters required for pharmacy when the length of the queue is less than or equal to one.

Table 8: The number of counters required for the pharmacy on weekdays

Hour	Mon	Tue	Wed	Thu	Fri	Average
8-9 a.m.	3	3	3	3	4	3
9-10 a.m.	5	5	3	5	3	4
10-11a.m.	3	3	3	4	4	3
11-12p.m.	3	3	3	3	3	3

Table 9: The number of counters required for the pharmacy on weekends

Hour	Sat	Sun
8-9 a.m.	2	2
9-10 a.m.	3	3
10-11a.m.	5	3
11-12 p.m.	5	3

According to the arrivals for the pharmacy suitable number of counters was calculated. Although there are three counters currently exist, according to the result, four counters are required in some days.

IV. CONCLUSION

According to the results, the number of doctors required for consultation and pharmacy can be changed hourly and daily or the average number of doctors can be assigned to work hourly for all days. The results of the pharmacy show that 4 or 5 counters are required to reduce the waiting time in some hours of some days.

According to the model outputs in each section, an arrangement can be proposed looking at the marginal changes for each part with respect to the average number in the queue and the average waiting time in the queue. One needs to be mindful of the cost involved in achieving these marginal changes. More doctors assigning to the roster cannot be cost effective, thus the balance between the number of doctors, costs, and optimal system performance is important for sustainability.

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94

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