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A low-cost polarimeter for an undergraduate laboratory to study the polarization pattern of skylight

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A simple, low-cost, fully automated polarimeter, which demonstrates fundamental properties of skylight scattering and polarization for undergraduate physics students, is described. The polarimeter includes a microprocessor-based control unit, a Sun tracker, an elevation-azimuth mount with two degrees of freedom, and a polarization sensor unit equipped with a light-dependent resistor for measuring light intensity. Results obtained in the principal plane of the Sun using the polarimeter on a relatively clear day, together with the theoretically expected results for a molecular atmosphere, are presented. A root-mean-square error comparison indicates fairly good agreement between theory and experiment. Construction and experimentation with the polarimeter will provide students with insight into important physical concepts involved in skylight scattering and polarization as well as improve their instrumentation capabilities. © 2017 American Association of Physics Teachers. [http://dx.doi.org/10.1119/1.4971159]

I. INTRODUCTION

Polarization of light is an important fundamental undergraduate-physics concept that is useful in understanding various natural phenomena and applications.¹ For example, some insects utilize polarized light for navigation, and the polarization pattern of skylight is employed in atmospheric optics techniques.^{2–8} In order to consolidate their understanding of polarization effects, students need to possess a good understanding of the underlying principles as well as acquire hands-on experience. Although the former objective can be achieved through theory courses in optics, students do not generally get an opportunity to verify polarization theory experimentally as the polarimeters employed for such purposes are rather expensive. This paper describes a low-cost polarimeter, designed for the above-described educational purpose. This polarimeter is appropriate for an undergraduate student's senior-year project and can be employed to demonstrate most of the fundamental properties of the polarization pattern of skylight.

For over two centuries, experimental investigations on the polarization of skylight have been carried out by various scientists, including Rayleigh, Brewster, and Babinet.^{2,6}

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electronic detectors, which have the advantage of giving quantitative data with a higher accuracy.^{6,9–11} All of these early polarimeters were used to scan the sky, particularly concentrating on the principal plane of the Sun, which is the vertical plane at the observation site that passes through the Sun.¹² Present-day studies on skylight polarization are carried out using full-sky imaging video polarimeters equipped with digital sensors and fish-eye lenses, which can take a picture of the whole sky at any given instant. The resulting recorded data can be analyzed using image processing techniques to find the polarization pattern of the entire sky hemisphere.^{5,13} These state-of-the-art polarimeters have the advantage of obtaining a snapshot of the entire sky in a single image so that the variation of the solar position with time does not affect the conclusions drawn using these data. In contrast, data obtained using the previously employed method of scanning the sky are acquired over (at least) a few minutes, during which time the Sun's position changes. However, given the high cost of state-of-the-art polarimeters,

Initially, visual polarimeters equipped with dichroic polar-

izers were employed to investigate the polarization pattern of skylight. With advances in physics and electronics, visual

polarimeters were replaced by units with photomultiplier and

the training required to properly operate these instruments, and the advanced knowledge needed to process their data, work with such equipment is beyond the reach of most undergraduates. Therefore, there is an educational need for low-cost polarimeters such as the one described in this paper. This instrument is specifically designed for undergraduates to use to verify the basic physics concepts involved in scattering and polarization of skylight as well as to explore the influence of atmospheric turbidity on the polarization pattern. The low cost of this instrument is made possible by the inexpensive light-dependent resistor (LDR) now widely available commercially.

In Sec. II, basic concepts required to understand the polarization of skylight are briefly outlined. Construction and operation of the polarimeter are described in Sec. III. Experimental results obtained with the polarimeter are presented in Sec. IV, together with a comparison of these data with the polarization pattern expected theoretically. Section V gives a few concluding remarks about the low-cost polarimeter and some suggestions for possible further extensions.

II. THEORY OF SKYLIGHT SCATTERING AND POLARIZATION

Earth's atmosphere consists of air molecules, which are distributed and oriented randomly. When unpolarized sunlight enters the atmosphere, these incident light rays cause air molecules to act as dipole oscillators in a plane normal to each incoming ray. These molecular oscillators emit electromagnetic radiation in all directions, except along their axes of oscillation, a process that is well described by Smith in terms of the electromagnetic fields.³ This re-emission and redistribution of electromagnetic radiation by air molecules is known as *scattering*.

In this paper, the following simple explanation of the polarization of light due to scattering will be sufficient to explain our experimental results. In the process of scattering by atmospheric gas molecules, unpolarized sunlight is transformed into partially polarized light, where the extent of partial polarization depends on the direction of scattering. This process is illustrated in Fig. 1, which shows a single scattering event in the principal plane of the Sun. In this figure, the transverse electric field E_0 of the unpolarized incident light is represented as a circle with vectors of equal length in two perpendicular directions. After being scattered by scattering angle θ with respect to the incident direction, the electric field of the resulting partially polarized sunlight is represented by an ellipse of major axis E_0 and minor axis $E_0 \cos \theta$, where both of these axes are transverse to the new propagation direction. This ellipse can be considered as the projection of the circle representing the unpolarized light (whose plane is perpendicular to the incident ray) along the polarization plane of scattering (which is perpendicular to the scattered ray). This projection reduces to a line corresponding to linearly polarized light when viewed at a scattering angle of 90°. Along an arbitrary viewing direction, the major axis of the ellipse represents light polarized in the direction normal to both the scattered ray and the principal plane (which is the plane of the page in Fig. 1) and this major axis has the same magnitude E_0 as the incident light. The minor axis of the ellipse represents light polarized in the direction normal to the scattered ray and lying on the principal plane and it has a reduced magnitude $E_0 \cos \theta$.

Because the scattering centers (i.e., air molecules) are randomly distributed in the atmosphere, the total irradiance is obtained by adding the individual irradiances of the two polarizations, rather than adding the electric fields and then squaring this sum to determine the total irradiance. That is,



Fig. 1. Unpolarized sunlight being polarized by an atmospheric gas molecule in the principal plane of the Sun, where θ_{sun} and θ are the Sun's elevation angle and the scattering angle, respectively.



defining $I_1 = E_0^2 \cos^2 \theta$ and $I_2 = E_0^2$ to be the irradiance in the principal plane and the plane normal to the principal plane, respectively, the total irradiance is $I_{\text{tot}} = I_1 + I_2$.

The *degree of linear polarization* d_l is defined as the ratio of the irradiance due to linearly polarized light to that of the total irradiance. Skylight, which in general is partially polarized, may be considered to be a combination of "natural" (unpolarized) sunlight and linearly polarized light.^{3,6} Then, by modeling the air molecules as dipole oscillators under the influence of the electromagnetic field of the skylight, d_l can be obtained as^{3,6}

$$d_l = \frac{|I_2 - I_1|}{|I_2 + I_1|} = \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta}.$$
 (1)

This relation can also be written as

$$d_l = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}},\tag{2}$$

where I_{max} and I_{min} are the maximum and minimum values of the irradiance due to polarizations in the principal plane and the plane normal to the principal plane, respectively. This expression is the same as the theoretical expression obtained for a molecular atmosphere using the electromagnetic dipole oscillator model.³

Experimentally, the degree of linear polarization can be determined for a given observation direction by using a linear polarizer placed in front of a detector sensitive to light intensity. The sensor used in our setup is a light-dependent resistor (LDR), a device whose resistance depends on the irradiance *I* of the incident light. When connected to appropriate excitation circuitry, the voltage across the LDR varies proportionately to *I*. Then, by pointing the detector in a given direction and rotating the polarizer in front of it, the maximum and minimum voltages across the LDR are recorded, allowing d_I to be computed using Eq. (2).

III. CONSTRUCTION AND OPERATION OF THE POLARIMETER

Our fully automated polarimeter consists of four main components: control unit, tracker, elevation-azimuth mount, and polarization sensor unit. The function of each component and a brief description of the construction are given in this section.

A. Control unit

The control unit oversees the operation of every electronic component in the polarimeter. This unit employs a PIC 16F887 microcontroller (Microchip Technology, Inc.) and its controlling program is built using the MIKROC PRO software package.¹⁴ A flow chart illustrating the main functions of the polarimeter is shown in Fig. 2, and the pin selection of the circuit board for various functions of the polarimeter is illustrated in Fig. 3. The driver circuits and the PCB designs for the circuits were constructed using PROTEUS 7 software.

The control unit carries out several calculations during its operation such as calculating the degree of linear polarization, the Sun's angle, and the current position of the sensor unit. This information is displayed on an LCD panel controlled by the microcontroller.¹⁵ Finally, the microcontroller



Fig. 2. Flow chart of the control unit functions.

sends these data as a data packet through RS232 protocol to the computer, where the data are stored for later retrieval.

The polarimeter uses two unipolar stepper motors and one bipolar stepper motor for vertical and horizontal rotations of the polarizer unit and the rotations of the polarizer around the collimator axis.^{16–18} Vertical rotations (around a horizontal axis) are activated by a unipolar stepper motor connected to the polarizer unit through two gear wheels, while horizontal rotations (around a vertical axis) are activated with the aid of a vertical axle connected to a stepper motor through two more gear wheels.¹⁹ The gear wheels ensure smooth rotations of the rotating structure and stability against factors such as wind.

B. Sun tracker

The Sun tracker rotates the collimator tube automatically towards the Sun with the aid of two window-comparator circuits, each one consisting of two LDRs (X_1 , X_2 and Y_1 , Y_2) operating based on the window comparator method. One comparator circuit is responsible for finding the Sun's position with respect to the vertical plane, while the other circuit is

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Fig. 3. Pin selection of the control unit circuit board.

responsible for doing so with respect to the horizontal plane. The arrangement of LDRs in the window comparator is shown in Fig. 4(a). Figure 4(b) shows the Sun tracker with the positioning of the 5-mm LDR sensors (manufactured by Excelitas Technologies, Inc.), which were partitioned by walls of black cartridge paper inclined at 90° angles. The separation between adjacent sensors is about 1 cm.



Fig. 4. (a) Arrangement of LDR sensors in the window comparator circuit; (b) Sun tracker.



Fig. 5. Schematic diagram of the window comparator circuit.

Each window comparator circuit consists of two outputs. With this arrangement, when the sensor unit is properly aligned towards the Sun, each pair of LDRs receives the same irradiance and both outputs of the comparator circuit achieve high state values (logic 1). All four outputs of the sensor unit reach the high state only when the sensor unit is properly aligned with the Sun both horizontally and vertically. The four outputs of the sensor unit are connected to four digital (I/O) pins of the microcontroller. The window comparator circuit was built using an LM 324 single supply op-amp¹⁶ manufactured by Texas Instruments, Inc., and is shown in Fig. 5. The microcontroller keeps rotating the elevation-azimuth mount until it reads the desired output state of the sensor, indicating the correct alignment with the Sun.

C. Elevation-azimuth mount

Once the Sun tracking is complete, any rotation around the horizontal axis scans the principal plane of the Sun. Figure 6 illustrates the rotations of the polarimeter with two degrees of freedom. The microcontroller is programmed in such a way that the collimator returns to a horizontal position in the principal plane after tracking the Sun. Thereafter, its position is incremented by 2° at a time with the aid of a stepper motor around the axis of the collimator and is able to



Fig. 6. Rotating structure with two degrees of freedom.

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Fig. 7. Schematic diagram of the LDR sensor circuit.

cover nearly 180° in the principal plane of the Sun. The structure of the elevation-azimuth mount is assembled using industrially available 8 mm aluminum thread bars. The polarimeter has overall dimensions $18 \text{ cm} \times 25 \text{ cm} \times 36 \text{ cm}$ and weighs about 1 kg.

D. Polarization sensor unit

The polarization sensor unit, consisting of a linear polarizer and a light detecting circuit, is used to measure the degree of linear polarization of skylight. The light sensitive circuit consists of a 5-mm LDR (manufactured by Excelitas Technologies, Inc., model number VT935G), an LM 324 single supply op-amp (manufactured by Texas Instruments, Inc.), and a 2-k Ω variable resistor. The LDR and the resistor are connected in series as shown in Fig. 7 and the output voltage across the LDR is determined according to the voltage division rule between two resistors. As a result, the output voltage varies in the range 0-5 V. The variable resistor is adjusted so as to have the maximum output voltage range between the maximum and the minimum intensity levels. The output of the circuit is connected to the RA0 pin (analog I/O pin) of the microcontroller through the op-amp, which is configured as a voltage follower to provide impedance matching between the sensor output and the microcontroller. The analog-to-digital converter (ADC) of the PIC 16F887 microcontroller is used to measure the output voltage of the sensor.¹⁴ The PIC 16F887 has a 10-bit ADC; that is, it has $2^{10} = 1024$ quantization levels with a sensitivity of 5 V/ 1024 = 0.0048 V in the sensor voltage output. The response time of the sensor unit is 50 ms and it requires about 10 min to obtain a complete set of measurements for a given position of the Sun.

A stainless steel tube of diameter 7 mm and length 10 cm serves as the collimator. When the collimator is exposed to skylight, a light cone with an apex angle of 4° (2° above and 2° below the sun) is incident on the LDR detector. The incident light is linearly polarized parallel to the transmission axis (TA) of the polarizer. As the polarizer is rotated around the axis of the collimator (with the aid of another stepper motor), this light intensity reaches a maximum I_{max} when the TA is parallel to the major axis of the polarization ellipse of the incoming skylight and becomes a minimum I_{min} when the TA aligns with the minor axis of the polarization ellipse of the incoming skylight. Intensities I_{max} and I_{min} are proportional to the voltages measured for each of the above cases



Fig. 8. Image of the polarimeter.

and the degree of linear polarization is determined using the measured maximum and minimum voltages for I_{max} and I_{min} in Eq. (2). Figure 8 shows an image of the polarimeter.

Nonlinear response of the detector

The response of the LDR detector used is not linear, particularly for largely varying intensities such as those observed directly below the Sun and in directions away from the Sun. However, this nonlinearity does not affect our computations such as determining the position of the maximum degree of linear polarization graphically, which is a turning point (maximum) of the graphs, although it could affect the degree of polarization.²⁰ In this work, no correction is made to the nonlinearity of the detector, and the voltage change is assumed to be proportional to the light intensity change.

E. Operation of the polarimeter

Initially, the polarimeter is turned towards the Sun manually so as to receive sunlight from that direction. Next, the Sun tracker is turned on so that the receptor is automatically and accurately oriented in the direction of the Sun. When the polarimeter is now rotated around a horizontal axis, the receptor scans the Sun's principal plane. All angle measurements are taken on this plane, scanning as much of the upper hemisphere as possible at 2° intervals. At each angle setting, the polarizer is rotated through 180° in a plane normal to the principal plane at 2° intervals and the maximum and minimum voltages observed across LDR are recorded. With these measurements, the degree of linear polarization is then computed. Graphs of the variation of the degree of linear polarization with the angular solar distance reveal various features of skylight polarization.





Fig. 9. Measured (markers) and theoretical (curves) values of the degree of linear polarization (d_i) as functions of the angular solar distance. Measurements were made on February 7, 2014 in the suburbs of Colombo (Latitude: 6.93°, Longitude: 79.93°, and Altitude: 33 m).

IV. RESULTS AND DISCUSSION

In this section, data taken using the polarimeter are presented, together with the theoretical expectations based on Rayleigh theory. All data were collected on February 7, 2014 at a location in Mulleriyawa New Town, (Latitude: 6.93° ; Longitude: 79.93° ; Altitude: 33 m) in the suburbs of Colombo. Experimental results are shown using different point markers and the exact times of commencing measurements are indicated in the inset of Fig. 9. Also shown as smooth curves are theoretical results for a molecular atmosphere based on the following relation, which is a re-written form of Eq. (1)

$$d_l = d_{lmax} \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta}.$$
(3)

The scaling factor d_{lmax} in Eq. (3) is used in order to obtain a good fit with the experimental results. This coefficient depends on many parameters such as the high optical depth resulting from the long air column at the measurement site (which is not far above sea level), the elevation of the Sun, and the prevailing atmospheric conditions (including the particulate matter present in the vicinity of the measurement site).^{8,21,22} The best fit between theory and experiment was computed by varying d_{lmax} to obtain the minimum value of the root-mean-square (rms) error (difference between theoretical and experimental results for d_l), to the nearest third decimal place. This value ranged between 0.014 and 0.016 for the three sets of measurements reported in Table I, indicating an average error of about 1.5% in the degree of polarization.

All measurements indicate a gradual increase in the degree of linear polarization with increasing angular distance from the Sun, with a maximum around 90° and a gradual decrease

thereafter. The theoretical degree of linear polarization for (small) angles near the Sun should be zero for a molecular atmosphere. However, values read from our plots range from 0.025 to 0.035. One reason for this deviation of the smallangle experimental results from theoretical expectations may be due to the nonlinear response of the LDR detector for the very high intensities directly below the Sun. Another possibility is multiple scattering events enhanced by the presence of aerosols not considered in the theoretical expression used in this work. In most of the investigations of skylight polarization carried out using full-sky imaging video polarimeters, angles near the Sun are blocked in order to protect the cameras from the highly intense direct sunlight.⁵ Apart from this, a correction was made to Eq. (3) because the data taken at 08:15 h indicated an initial error of 2°. Thus, the equation $d_l = d_{lmax} \times [1 - \cos^2(\theta + 2^\circ)] / [1 + \cos^2(\theta + 2^\circ)]$ was used to plot the theoretical graph at 08:15 h. Similar deviations have been observed in skylight polarization experiments in the solar almucanter.⁷ The slight deviations in each of the experimental plots from a smooth curve could be the result of occasionally passing clouds and the small vibrations that could be observed in the polarimeter during its rotations.

The results summarized in Table I clearly confirm that the maximum polarization is observed at a scattering angle of almost 90° and that the polarization pattern is well described by the theoretical results for a molecular atmosphere described by Rayleigh's single-scattering theory, together with a simple scaling factor d_{lmax} .

The maximum degree of linear polarization values close to 40% in the three results differs from the 100% value expected from single-scattering Rayleigh theory for many reasons. Major sources of this difference are the multiple scattering events among the atmospheric gas molecules due to the long air column at the measurement site, which is not far above sea level, and the presence of aerosol (turbidity) in

Table I. Summary of the experimental results along with the rms error in the degree of polarization computed by comparing with theory.

Time	Elevation of the sun in the principal plane (θ_{sun})	Maximum degree of polarization (d_{lmax})	Scattering angle at maximum degree of polarization (θ_{max})	rms error in d_i
07:30 h	19°	38.4%	90°	0.015
07:45 h	22°	37.8%	90°	0.014
08:15 h	30°	36.8%	88°	0.016

the atmosphere. Both of these effects enhance multiple scattering, which tends to depolarize skylight, resulting in a lower value for d_{lmax} than that predicted by the Rayleigh theory used in this work.^{22,24} Additionally, the elevation angle of the Sun affects the maximum degree of linear polarization, which can be seen in the slight variations between 36.8% and 38.4% in the results for a difference in the solar elevation angle from 19° to 30° .

Many researchers have investigated the relationship between the maximum degree of linear polarization and atmospheric turbidity. These workers introduced semi-empirical formulas, which include a turbidity factor, to describe the resulting skylight polarization pattern. One such formula is due to D. G. Stamov, where the turbidity factor is indirectly included through the maximum degree of linear polarization⁸

$$d_l = \frac{d_{lmax}\sin^4\theta}{1 - d_{lmax}\cos^4\theta}.$$
(4)

When our experimental data plots were compared with those produced using the semi-empirical Stamov formula [Eq. (4)] instead of the single-scattering Rayleigh formula [Eq. (3)], rms errors around 0.02 were found between theory and experiment. While the agreement was good for scattering angles between 60° and 120° , significantly larger deviations were noticed for other scattering angles compared to graphs plotted using Eq. (3). In order to ascertain whether this is due to errors in our data resulting from disregarding the nonlinearity of the detector or whether Eq. (3) is sufficient to describe the skylight polarization pattern under the atmospheric conditions prevailing at the measurement site, further investigations are needed. We avoid a lengthy discussion on this issue since it is beyond the scope of this paper. Readers interested in this issue should refer to Refs. 7, 8, 22,23 and 25.

V. CONCLUSION

Experiments carried out using the polarimeter will provide physics students with insight into the physics of skylight scattering and polarization as well as provide a basis for understanding related phenomena. In addition, construction of this instrument will also give these students an opportunity to apply and broaden their knowledge of electronics and microprocessors. We believe that this polarimeter, and variations of it, can be further improved to illustrate some of the finer features of the skylight polarization pattern. This further work may include investigating the wavelength-dependence of the maximum degree of linear polarization with the aid of color filters and determining the positions of the polarization neutral points resulting from multiple scattering events and their deviations from expected locations due to atmospheric turbidity.^{22,23} Such studies can lead to gathering information about atmospheric turbidity levels prevailing at the measurement site.⁸ Furthermore, the study can be extended to an observation of the polarization pattern of moonlight scattered by the atmosphere on full-moon nights with clear skies, as this pattern resembles that of sunlight.^{2,25} In measuring moonlight intensities, however, suitable detectors that are sensitive to low levels of light intensities would need to be employed.

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