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Research Note

ANOTHER METHOD FOR DRAWING AN ELLIPSE

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Abstract

A method of drawing an ellipse with instruments contained in a box of geometrical instruments, is described.

Key words: Ellipse

Running title: Another method for drawing an ellipse.

Introduction

The well known standard method of drawing an ellipse uses the property of the ellipse that $S_1P + PS_2 =$ constant, where S_1 and S_2 are the foci of the ellipse and P any point on it. If O is the centre of ellipse, $OS_1 = OS_2 = ae$; $e^2 = 1-b^2/a^2$, where a = semi major axis, b = semi minor and axis e = eccentricity of ellipse.

The construction procedure consists of tieying a string of length 2 (a+ae) into a loop and making it encircle two pins stuck at S_1 and S_2 and a pencil. The pencil is made to pull the loop so that the loop becomes taut and forms a triangle PS_1S_2. Now P is moved keeping the string pulled taut against the pins and pencil all the time, to draw the ellipse (Fig. 1)

The method to be discussed below has the advantage that no estimate of 'e' need be made and it requires only a right angle corner (which could be cut-off from a piece of thick paper) and one of many devices available to draw a circle. All requirements are found in a box of geometrical instruments available to any school going child.

Procedure

To draw an ellipse of semi major axis 'a' and semi minor axis 'b' the following steps are gone through. Refer Fig. 2.

Steps

1. Draw a circle of radius "a"
2. Construct a chord of the circle of size "2b"
3. Mark the centre of this chord, N.
4. Place a set square APB (whose $\widehat{APB} = 90^0$) so that P lies on the circumference of the circle and AP passes through N.
5. Draw PB.
6. Move P to different positions on circumference and repeat steps 4 and 5 when half the ellipse will be hatched out.
7. Flip over the set square and repeat above with the remaining half of circle when the rest of ellipse will be hatched out.

The above procedure is greatly facilitated if a pin, or better, a drawing pin is stuck at N to act as a guide along which the side PA can slide.

Mathematical proof of construction

Equation of circle of radius 'a' is $x^2 + y^2 = a^2$.

The coordinates of the point N on chord (Fig. 2) is (-p,o) where $b^2 = a^2-p^2$

and 2b = Chord length

The equation to normal of NP_1 is

$$y = -x (x_1 + p) /y_1 + (px_1 + a^2) /y_1$$

By rearranging the above equation we get,

$$yy_1/(px_1 + a^2) + x(x_1 + p) / (px_1 + a^2) = 1$$

Multiplying numerator and denomenator of the first term of the above equation by A^2 and the second term by B^2 respectively,

$$[x(x_1 + p) A^2/ (px_1 + a^2)]/A^2 + [(yy_1B^2) / (px_1 + a^2)] /B^2 = 1$$

where, $X_1 = (x_1 + p) A^2 / (px + a^2)$ and $Y_1 = y_1B^2/(px_1 + a^2)$

The equation now becomes

$$xX_1/A^2 + yY_1/B^2 = 1$$

This is a tangent at (X_1, Y_1) to the ellipse

$$x^2/A^2 + y^2/B^2 = 1$$

Hence the normals from $(x_1, y_1), (x_2, y_2)$.... on the circumference of circle will touch a common ellipse at single points (X_1, Y_1), (X_2, Y_2) ie the normals will envelope this ellipse.

In the construction we choose that A=a the radius of circle, B=b the semi-chord NP_1 making the ellipse

$$x^2/a^2 + y^2/b^2 + 1$$

Bold line ellipse

To reduce the hatched area visible after ellipse is constructed, the normal need be made visible only from the (upper/lower) side of the rectangle upto the circle circumference. In fact, with practice a bold line ellipse can be obtained by making only very short lenghs of the normals visible, starting from the minor and major axes positions which are known.

Illustrations

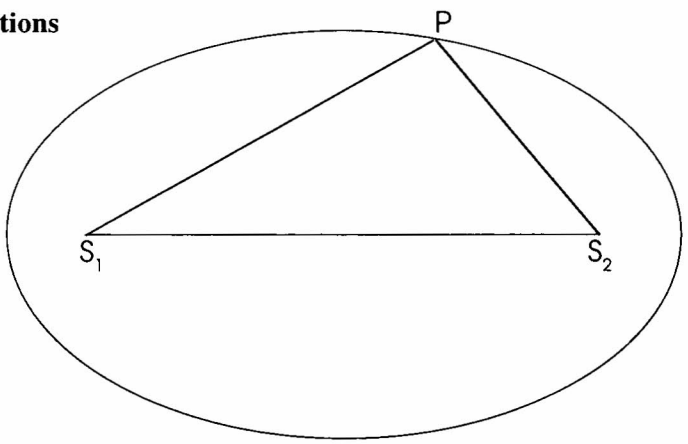

Fig. 1 The conventional construction of an ellipse using a looped string.

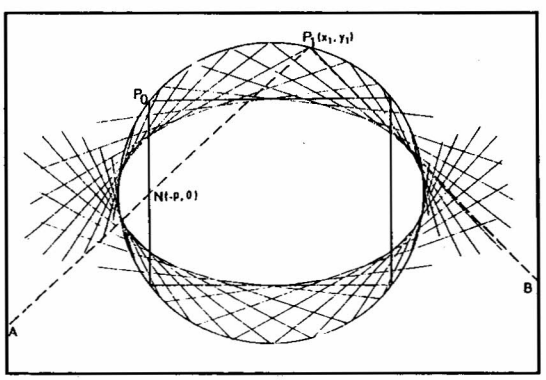

Fig. 2 An alternative method of constructing an ellipse.