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VIBRATIONAL MODES OF A RABANA (DRUM)

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Abstract

The vibrational modes of a "Virindu" Rabana membrane (of diameter 13 inches), which was driven by a loudspeaker, were studied by using a microphone. The vibrational modes were observed at frequencies below 500Hz. The nodal patterns seen here were slightly different from theoretically expected patterns for an ideal membrane. It was observed that most of the vibrational modes were degenerate. These degeneracies could be due to the non-uniformity of the membrane and the imperfect membrane boundary. However, the observed frequencies were close to that obtained from theory.

Key Words: Drum- Rabana, Vibrations

Introduction

Drums have been used for a long period of time in the history of mankind. They can give wide range of sound signals depending on the nature of the instrument used as well as how it is been played. Basically, a drum consist of a membrane(s) and a frame to which the membrane(s) is(are) fixed. Most drums consist of a frame made of wood or metal and membranes made of animal skin or any other synthetic materials. Drums can be divided in to three categories[1] according to their vibrating system.

1. Those consisting of a single membrane which is coupled to an enclose air cavity such as "Tabla" and Thammattam".
2. Those consisting of a single membrane which is opened to the air at both sides such as "Rabana".
3. Those consisting of two membranes coupled to an enclosed air cavity such as "Uddakkiya" and "Dowla".

The 'Rabana" which belongs to the second category mentioned above is typically made of a goat skin which is coupled to a conical wooden frame so that it is under tension. The wooden frame of "Rabana" is made by a hard wood like Jak. Usually, the edge of wooden frame to which the membrane is flxed is almost circular. The "Rabanas" in Sri Lanka are divided in to two categories known as "Virindu" and "Banku" (bench), depending on their sizes. While the diameter of a 'Virindu Rabana' is about eleven to fifteen inches, the diameter of a "Banku Rabana" is typically larger than twenty three inches.

In this experiment, a "Virindu Rabana", of diameter of thirteen inches was studied. The vibrational modes of the "Rabana" were identified by observing the maxima found in the sound signal produced and the nodal lines on the membrane of "Rabana". The observed frequencies were compared with the theoretical frequencies for vibrational modes of a circular membrane.

2. Theory

The vibrational modes of a circular membrane have been studied in detail and are described in standard text books in Physics and related fields [2,3,4] The equation for a displacement $U(r,\theta,t)$ of the circular membrane, from its equilibrium position is given in polar coordinates (r,θ) and time t

$$\frac{\partial^2 U}{\partial t^2} = \frac{s}{p}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}\right\} \quad \text{---} (1)$$

where s and p are tension and mass per unit area of membrane, respectively. The solution $U(r,\theta,t)$ is given by

$$U(r, \theta, t) = A \exp(\pm i\omega t)\exp(\pm im\theta)J_m\left(r\omega\sqrt{p/s}\right)$$

where ω is the frequency associated with the vibration

and A is an arbitray constant with m= 1,2,3,..

and J_m are known as Bessel functions.

However,

$$J_m\left(r\omega\sqrt{p/s}\right) = 0$$

at r = a (the radius of the circular membrane). This gives the normal frequencies

Table I. The first few positive roots of Jm (x)=0. Note that for all cases listed successive large roots differ approximately by π=3.1459... (*Adapted from* ref. 5)

m=0	m=1	m=2	m=3	m=4	m=5	m=6
2.4048	3.8317	5.1356	6.3802	7.5883	8.7715	9.9361
5.5201	7.0156	8.4172	9.7610	11.0647	12.3386	13.5893
8.6537	10.1735	11.6198	13.0152	14.3725	15.7002	17.0038
11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208
14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861
18.0711	19.6159	21.1170	22.5827	24.0190	25.4303	26.8202

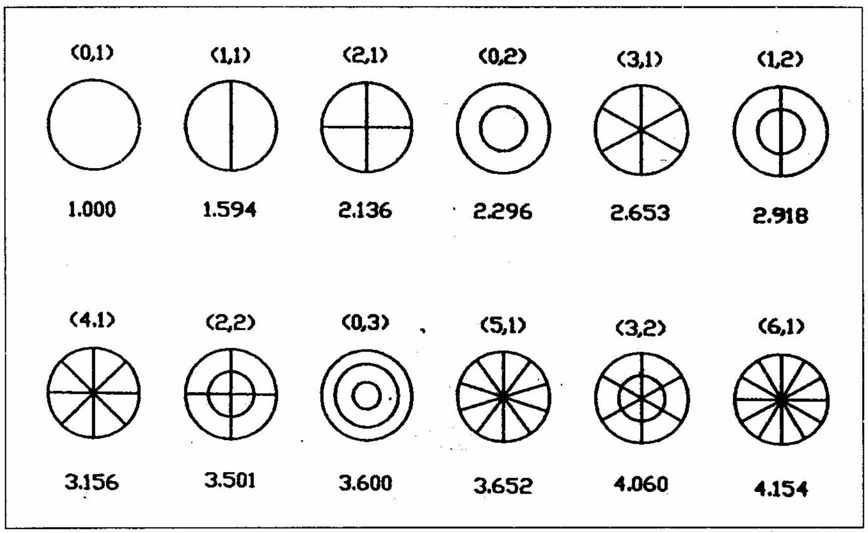
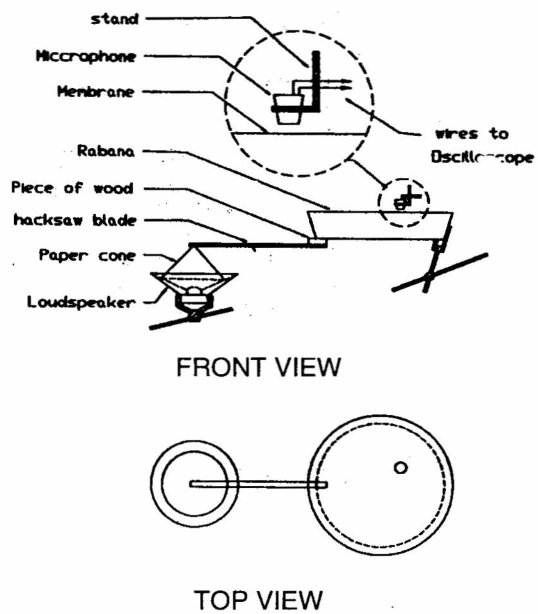


Figure 1. The nodal patterns of the first 12 vibrational modes of an ideal circular membrane while each mode is designated by (m,n), the relative frequency of each mode is given its beolw its nodal pattern.
(Adapted from ref. 1)

of the system and Table I and figure 1 show the frequencies[5] and their respective vibrational patterns[1]. In order to convert these to actual frequencies they have to be multiply by $(2.405/2\pi a)\ \sqrt{S/p}$

3. Experimental Method

In order to study the normal modes of the 'Rabana' membrane, the vibrations of an eight inch (3 Watts) circular loudspeaker was fed to the



Figur 2. The Schematic diagram of the mechanical system used to study the normal modes of the "Rabana"

"Rabana" via a lever mechanism. The mechanical system used is shown in Figure 2. A sinusoiodal signal which was produced by a signal generator was fed to the loudspeaker. The paper cone which was in direct contact with the loudspeaker foil transferred the vibrations to the "Rabana". via a hacksaw blade. When the frequency of the signal produced by the generator was equal to that of a normal frequency of the membrane of the "Rabana", it gave an appreciable vibration. This mechanism used was very effective in transferring the vibrations to the "Rabana" membrane and was adapted after trying out many other methods.

A microphone was mounted on a stand such that it could be moved parallel to the membrance of the "Rabana" The distance between the microphone and the membrane of the "Rabana" was kept at a distance of one inch. The amplified output signal from the microphone and the applied signal to the loudspeaker were simultaneously observed on an oscilloscope and the frequency of the applied signal was determined with a frequency counter.

The microphone was held above the membrane and the frequency of the applied signal was varied continously until the amplitude of the output signal from the microphone went through a maximum. This gives a particular normal frequency of the "Rabana". Subsequently, the microphone was moved parallel to the membrane while monitoring the output signal and observing the points on the membrane which gave maximum and zero signals. The vibrational patterns were identified in this manner. Thereafter, the microphone was held at the point which gave the maximum amplitude and the frequencies at which signal reduced by three and six decibels were identified for a particular mode by varying the frequency of the applied signal.

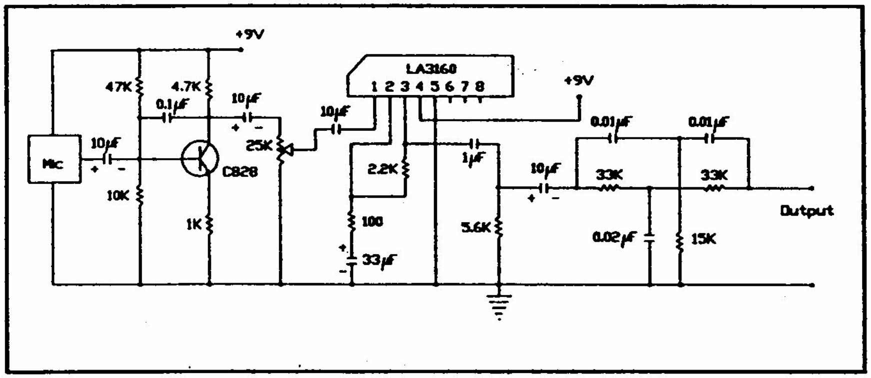


Figure 3. The schematic diagram of the circuit used to amplify the signal produced by the "Rabana" membrane

In order to observe the signals from the microphone, it was necessary to amplify it. Figure 3 gives the schematic diagram of the circuit used to amplify the signal, whcih had an amplification factor of about 25. This circuit consist of a low frequency pre-amplifier made out of the transistor C828, an amplifier made of IC LA3160 and a twin T filter capable of suppressing the signal at 50Hz.

4. Observations

The output signal was appreciably large at resonance frequencies compared

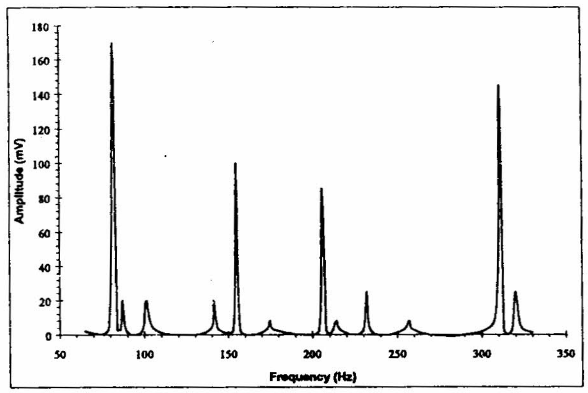


Figure 4. The observed variation of the maxima of the signal amplitude as a function of frequency at room temperature of 28^0 C and relative humidity of 62%.

with the rest. Figure 4 shows the variation of the maxima of the amplitude of signal produced from the vibrated "Rabana" membrane as a function of the frequency, when the air temperature was 28^{0}C and relative humidity was 62%. The corresponding nodal patterns observed are given in Figure 5. Table II gives the different modes (when identified) and the corresponding experimental frequencies for the above run.

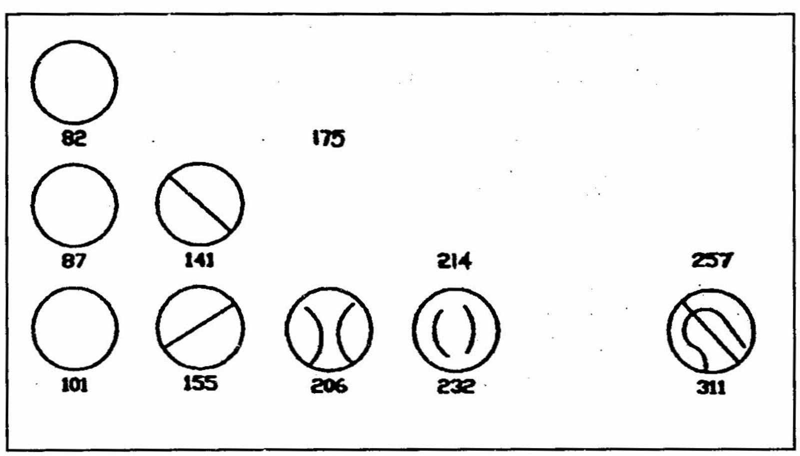


Figure 5. Observed nodal patterns corresponding to the signal in figure 4. The frequencies are given below each pattern.

Table II The comparison of observed frequency with the theoretical frequencies for an experiment performed at air remperature of 28^{0}C and relative humidity of 62%. The theoretical frequencies for $J_{0,1}$ is taken to be same as those of experiment.

Mode	Experimental frequency	Theoretical Frequency	Percentage deviation
$J_{0.1}$	101	101.00	
$J_{1.1}$	155	158.00	-1.90
$J_{2.1}$	206	217.87	-5.45
$J_{0.2}$	232	234.19	-0.94
$J_{3.1}$	Not observed	270.61	
$J_{1.2}$	311	297.64	4.49
$J_{0.1}$	87	87.00	
$J_{1.1}$	141	138.68	1.67
	214		
	257		
J0.1	82		
	175		

When the experiment was repeated many times certain nodal modes were not always present indicating that some of the modes were not stable. However, there were few frequently present modes which could be identified clearly and these modes are

Table III Frequently present modes which could be identified clearly and their comparison with theoretical frequency. The theoretical frequencies for $J_{0,1}$ is taken to be same as those of experiment.

Mode	Experimental	Theoretical	Percentage
$J_{0,1}$	101	101.00	
$J_{1,1}$	159	158.00	0.63
$J_{2,1}$	212	217.87	-2.69
$J_{0,2}$	236	234.19	0.77
$J_{3,1}$	263	270.61	-2.81
$J_{1,2}$	308	297.64	3.51

given in Table. III. Here, the experimental values for normal frequencies, which were obtained using an average of six different runs, compare well with that of theory. *For all runs,* the theoretical frequencies were calculated by using $J_{0,1}$ mode values of experiment as references and using multiplication factors given in Figure 1.

5. Discussion

In this experiment only frequencies below 500 Hz were observed as normal modes. At lower frequencies it was easy to idenfify the nodal patterns, but when the frequency was increased, the patterns were complicated and the amplitude of the signal was weak.

Since the amplitude of the Output signal was appreciable at resonance frequencies it helped to identify the resonance frequencies of the membrane. However, in this study some modes with very faint signals could have been missed because it was difficult to study every point of the membrane continuously for all frequencies with a single microphone.

For most experimental observations there were repetition of the same pattern at slightly difference frequencies. This result is known as degeneracy[6] of vibrational modes.

Moisture can change properties like elasticity and mass per unit area of the membrane as the membrane is made of an animal skin. Hence, it is important to consider the moisture content at the experimental surrounding. In this experiment, it was observed on damped days, the vibration of the membrane was very weak. On those days, only one or two vibrational modes were observed. This can be due to variation of the tension of the membrane or due to variation of the mass per unit area of the membrane. When the measurements were done on different days, the relative humidity and temperature of the surroundings was almost the same but the fundamental frequency of the membrane and the amplitude patterns were slightly different. Hence it is clear that the vibrational property of the membrane was not only related to the humidity of the surrounding but also depend on other environmental conditions. When the experiment was done continuously for about four hours it was observed that the fundamental frequency varied by about one or two hertz, although the hunudity of the surrounding remained unchanged. It is possible that the concentration of the moisture in the membrane could change because of the vibrations or due to other reasons, hence, the normal frequency would also vary correspondingly. However, in this experiment direct relationship between the variation of frequencies of the membrane and the moisture concentration in the membrane was not conclusively established.

The observed nodal patterns are the same predicted that by theory for lower order modes of (0, 1) and (1, 1). However, the higher order nodal patterns show slight deviations from theory, and this may be due to assymetry of the system which is created in the process of the construction of the Rabana.

The theoretical normal frequencies were obtained by assuming that the tension s, and density p (in equation 1) were independent of r and θ. However, in practice the membrane of the "Rabana" is fixed to the wooden frame by using nails. It is most likely that tension in the "Rabana" membrane is a function of both r and θ. Similarly, as the membrane is made of animal skin it is possibly non uniform. Also, the membrane boundary is slightly imperfect as it is constructed by hand, using natural wood and also due to the distortions caused by drying of wood under may lead natural atmospheric conditions. These conditions may lead to the distortion of the observed complex vibrational patterns, and possibly to the degeneracy of the vibrational modes. A Manzer and H.J.T.Smith[7] have also observed similar results. They explain this to be caused by an imperfect membrane boundary.

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