

RELATIVE GEOID OF CENTRAL SRI LANKA

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Abstract

Relative geoid of central Sri Lanka has been calculated adding contributions due to the Bouguer anomaly and topographic features over the region. A Fourier technique which makes use of fast Fourier algorithms was used to obtain the topographic component instead of classical gravimetric approach which is based on Stokes-Helmet method. These computations show that the relative geoid of central Sri Lanka is entirely positive with a maximum of 5.9 m close to Nuwara Eliya.

Key words: Relative Geoid, Topography, Sri Lanka

1. Introduction

The Geoid is an important concept in geodesy and applied mathematics which has many practical applications. Normally geoid is defined as the equipotential surface of the earth which passes through the mean sea level. One can visualize the geoid over the continents as the mean level at which water would lie if imaginary canals have been cut through the continents. The geoid is considered as the level of zero height for all navigational and surveying purposes.

Issac Newton in the 18th century showed that the shape of the geoid is an ellipsoid of revolution slightly flattened at the poles considering the centrifugal force arising due to the rotation of the earth and also assuming the earth is in isostatic equilibrium. Later it was shown by various workers that this is only an approximation to the true picture of the geoid and its actual shape is much more complex (Rapp, 1986). However, the ellipsoidal figure suggested by Newton is now considered as a reference and is known as the reference ellipsoid or reference spheroid. The shape of the geoid is expressed in terms of

deviations from this reference figure. These deviations are in the order of tens of metres and are mainly due to the effect of the lateral mass anomalies inside the earth and major topographic features on the surface of the earth.

The nature of the geoid on a global scale has been determined using observations made on the small deviations of the orbital parameters of artificial satellites from direct radar altimetry measurements (GEOS3, SEASAT and GEOSAT missions) (Kaula, 1986). Geoidal undulation maps produced by the above method provides an excellent picture of the long wavelength features of the geoid. However, this method is not satisfactory in determining the local geoid of a region with rough topography and complex geology due to its inability to resolve short wavelength components. Even the most detailed high resolution geoidal models currently available, OSU 86 E and OSU 86 F (Rapp and Cruz, 1987) can not resolve features having wavelengths more than 100 km. Therefore one has to adopt the classical gravimetric approach in determining local geoids.

The classical gravimetric approach which is based on the Stokes-Helmet method assumes that the topographic masses on the earth are condensed to its surface (Moritz, 1980). The gravitational attraction of these condensed masses is added to the Bouguer anomaly and transformed into geoidal undulations by integration. Hipkin (1988) presented a more accurate method of determining local geoids in which he uses a contour integration method to determine the effect due to topographic masses instead of classical Stoke-Helmet condensation reduction. However, the contour integration approach is not computationally tractable even though it provides accurate results. Application of Fourier techniques which makes use of Fast Fourier Transform Algorithms (Cooley and Tukey, 1965) provides a much more elegant way of determining local geoids (Parker, 1971; Stewart and Hipkin, 1990).

This paper describes an attempt to compute the relative local geoid of central Sri Lanka based on the Fourier technique. Most of central Sri Lanka is covered with a mountain range which exceeds 1800 m in height. There is a negative Bouguer anomaly reaching a minimum of 50 m Gal over this region. Contribution to the local geoid from the topographic features and from the Bouguer anomaly were separately calculated and added together to obtain the relative local geoid of this region.

2. Theoretical aspects

As described in the introduction, the deviation potential, (the difference between potential at a point on the geoid and the potentials at the reference ellipsoid) can be written as the sum of the potentials due to sub-surface mass anomalies (V_b) and the topographic masses (V_T)

$$V = V_B + V_T$$

Since we are computing the local geoid of a relatively small area it is possible to neglect the curvature of the earth and use a rectangular Cartesian coordinate system. The relationship between the Bouguer anomaly, Δg , and the corresponding potential, V_B can be written in the following way by solving the Laplace equation in rectangular Cartesian coordinates,

$$V_B = F^{-1} \left(\frac{F(\Delta g)}{k} \right)$$

In the above expression $F()$ and $F^{-1}()$ stand for the Fourier Transform and Inverse Fourier Transform-respectively and k stands for the wavenumber. The deviation potential due to topographic masses V_T can easily be calculated by the Fourier method (Parker, 1971; Stewart and Hipkin, 1990),

$$V_T = 2\pi\rho G F^{-1} \left(\frac{F(h)}{k} + \frac{F(h^2)}{2!} + \frac{KF(h)}{3!} \dots \dots \right)$$

It can be shown that V_T can be calculated with adequate accuracy by only considering the first term of the above equation (Stewart and Hipkin, 1990). Since the potential gradient gives the gravity, a potential difference ΔV that causes an undulation N in the geoid can be written as,

$$gN = \Delta V$$

where, g is the average value of gravity over the distance N . since the value of N is of the order of tens of metres, it can be assumed that g is a constant throughout this height. Using the above equations N_T and N_B can be written as,

$$N_T = \frac{2\pi\rho G}{g} F^{-1} \left(\frac{F(h)}{k} \right)$$

$$N_B = \frac{1}{g} F^{-1} \left(\frac{F(h)}{k} \right)$$

Once N_T and N_B are calculated by wavenumber division in the frequency domain, they can be added to obtain the relative geoid.

3. Relative geoid of central Sri Lanka.

Figure 1 and 2 give the Bouguer anomaly and topographic maps of Sri Lanka used in the computation of relative geoid over its central part using the Fourier technique. Application of the Fourier technique requires gravity and

topographic data regularly sampled at the nodes of a rectangular grid. Gravity values over the sea area NE of Sri Lanka are not available. Gravity anomaly values at those grid points were obtained by the extrapolation of known values over the land area. However, this will not affect the final results as we smooth the anomaly to come down to zero at the edges of the rectangular grid for the reasons explained below.

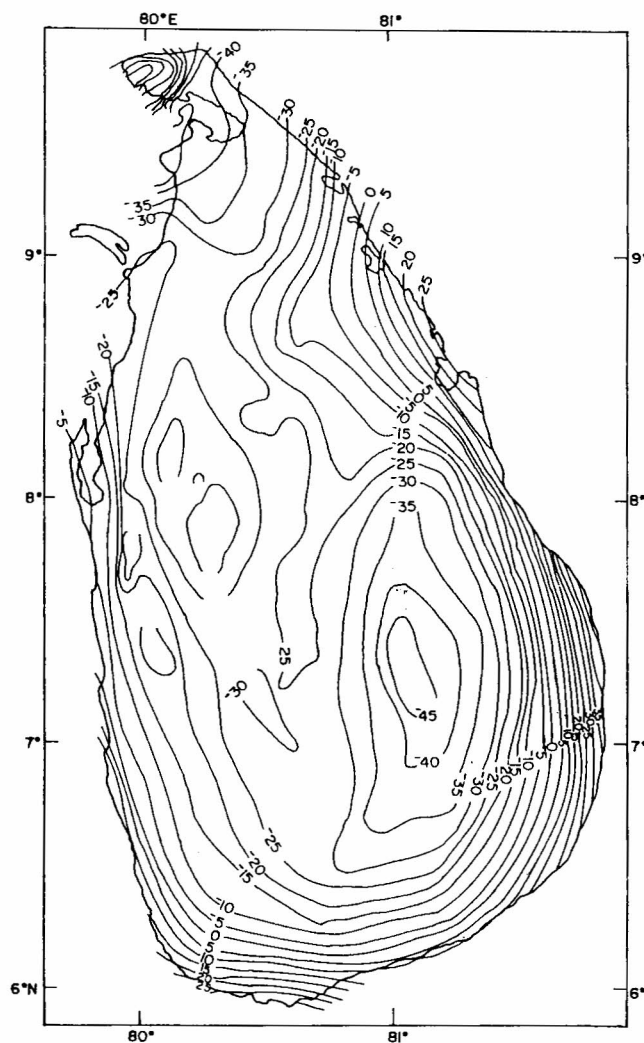


Figure 1. Bouguer anomaly map of Sri Lanka (redrawn from Hatherton et al., 1975).

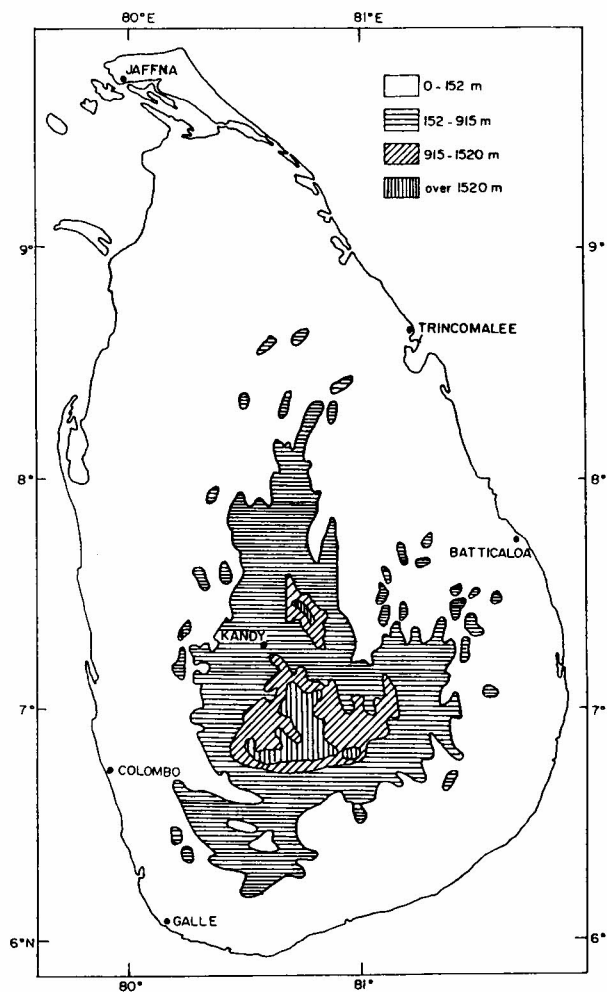


Figure 2. Topographic map of Sri Lanka (Redrawn from Hatherton et al 1975)

The Bouguer anomaly and the topographic maps of central Sri Lanka were digitized at 12 km intervals. Any regional trend present in the data were removed by subtracting a least square plane from the digitized data. Unlike the topography map, the Bouguer gravity anomaly map has sharp discontinuities at its edges. These sharp discontinuities in the spatial domain function give rise to spurious side lobes in the frequency domain function. Such spurious frequency components can be minimized either by cosine

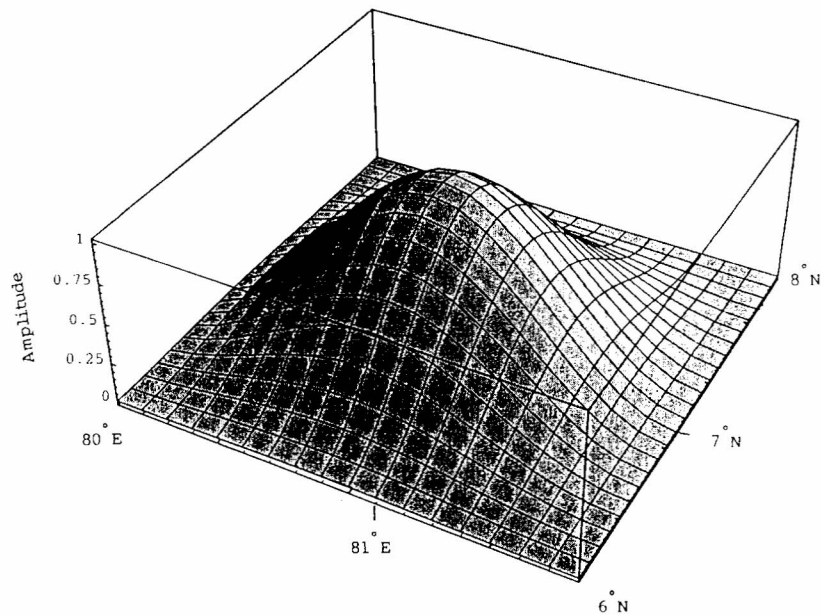


Figure 3. Two dimensional Hanning window through which the gravity and topographic data were passed before computation of their Fourier transforms.

tapering the data or by passing the data through a Hanning Window (Brigham, 1974). Gravity anomaly values used in this study were passed through a two dimensional Hanning window illustrated in Figure 3. As can be seen from Figure 3 Hanning filter smoothly reduces the anomaly to zero at the edges of the map avoiding sharp discontinuities. The Bouguer co-geoid and the contribution to the relative geoid from the topographic masses were computed separately and added together to get the relative geoid of central Sri Lanka (Figure 4, 5 and 6). The exact position of the zero height contour of the Bouguer co-geoid was determined by fitting a least square plan to the results obtained from the Fourier technique before adding two components together. The Bouguer co-geoid shown in Figure 4 has both negative and positive regions. However, over the central up country of Sri Lanka it is entirely negative and reaches a minimum of -1.1 m over Mahiyanganeya. On the other hand topographic component of the geoid is entirely positive and has a maximum of 6.7 m over Nuwara Eliya. Relative geoid of Central Sri Lanka which is a combination of two components mentioned above is also entirely positive having a maximum of 5.9 m close to Nuwara Eliya.

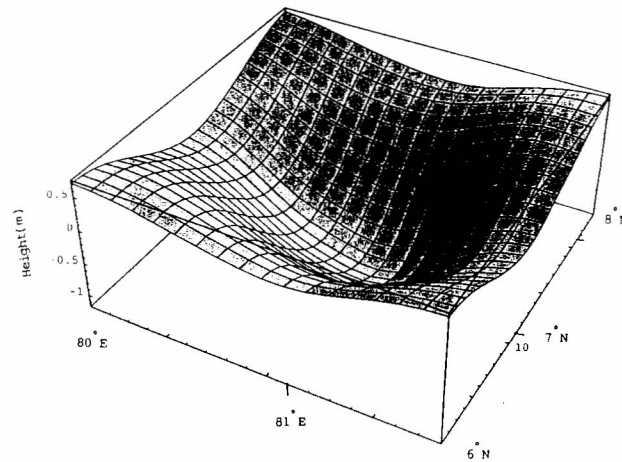


Figure 4. Bouguer co-geoid of Sri Lanka.

4. Discussion

The relative geoid of central Sri Lanka presented in this paper can be improved by several ways. As it is clear from Figures 4, 5 and 6 the component of the relative geoid of central Sri Lanka due to the topographic masses, which exceed 1800 m in height is much more prominent compared to the Bouguer component. Detailed topography of this region forms a complex pattern with several closely situated peaks and vallies. Therefore it will be a worthwhile exercise to recompute this component of the geoid using closely sampled topographic data. This will have the added advantage of minimizing the "corruption" of frequency domain information due to aliasing (Brigham, 1974).

Another weakness of the geoid depicted in Figure 6 is that it has been measured relative to an unknown plane. The absolute local geoid can be obtained by superimposing the local geoid on the global geoid computed from geopotential models such as OSU 86F (Rapp and Cruz, 1987).

As explained in section 2, calculation of the local geoid using the Fourier technique involves division of the Fourier transform of the topographic function and the Bouguer anomaly by the wavenumber. This division will upset the symmetry of Fourier transform making its inverse transform imaginary. Therefore it is necessary to rearrange the Fourier coefficients in such a way required symmetry is regained. This is a relatively straightforward operation in the one dimensional case. However, when dealing with two dimensional arrays it is necessary to carry out some tricky

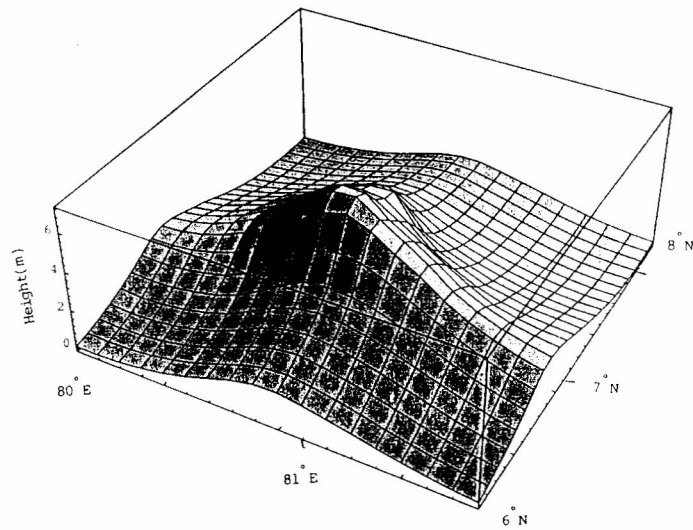


Figure 5. Contribution to the relative geoid of Central Sri Lanka from topographic masses.

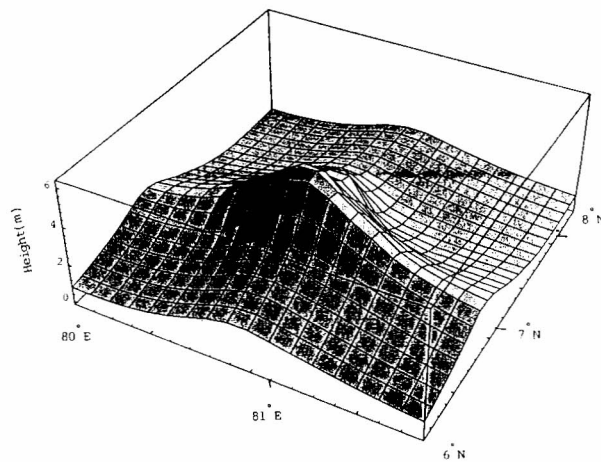


Figure 6. Relative geoid of central Sri Lanka.

manuring and should be done with care.

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References

- Brigham, E. O. 1974. The Fast Fourier Transform, *Prentice Hall, New Jersey*
- Cooley, J. W. and TUKEY, J. W. 1965. An algorithm for machine calculation of complex Fourier series, *Math, Comp.*, 19, 297-301.
- Hatherton, T., Pattiarachchi, D. B. and Ranasinghe, V. V. C. 1975. Gravity Map of Sri Lanka, 1:1000000, *Professional Paper No. 3, Geological Survey Department, Sri Lanka*.
- Hipkin, R. H. 1988. Bouguer anomalies and the geoid: a reassessment of Stokes' method, *Geophysical Journal*, 92, 53-66
- Kaula, W. 1986. Satellite Gravity, In *Methods of Experimental Physics, Geophysics*, edited by C. Sammis and T. Henyey, *Academic Press*.
- Moritz, H. 1980. Advanced Physical Geodesy, *Wichmann, Karlsruhe*.
- Rapp, R. H 1986. Status of the Geopotential, *Review of Geophysics*, 25 (5), 864-866.
- Rapp R. H. and CRUZ Y. J. 1986. Representation of the earth's gravitational potential in a spherical harmonic expression to degree 250, *Report No. 312, Department of Geodetic Science and Surveying, The Ohio State University*.
- Stewart, N. P. and Hipkin, R. H. 1990. A high resolution, high precession geoid for the British Isles, In *Sea surface topography and the Geoid*, Edited by H. Sunkel and T. F. Baker, 39-46, *Springer-Verlag, New York*.